Chapter 5 Nuclear Forces

5.1 Introduction

In Chapter 1, we discussed the four forces of nature, the electromagnetic, the strong (nuclear), the weak and the gravitational force. In dealing with the structure, reactions and decay of nuclei, we shall be dealing with the electromagnetic, strong and weak interactions. The principal force we shall concern ourselves with is the strong or nuclear force. In this Chapter, we shall summarize some important features of the nuclear force.

One basic characteristic of all the fundamental forces is their exchange character. They are thought to operate through the virtual exchange of particles that act as force carriers. What do we mean by the term virtual? We mean that the exchange particles only exist for a short time consistent with the Heisenberg Uncertainty Principle and cannot be detected experimentally.

How is this possible? Consider the familiar electromagnetic interaction. Two charged particles can be imagined to interact electromagnetically by the emission of virtual photons that are continuously emitted and absorbed by the particles (i.e., exchanged). The Heisenberg uncertainty principle tells us that

\[ \Delta E \cdot \Delta t \geq \hbar \]

or that we can “violate” the law of conservation of energy by an amount of energy \( \Delta E \) for a time \( \Delta t \) such that
\[ \Delta t = \frac{\hbar}{\Delta E} \]

(The emission of a virtual photon by a charged particle violates the law of conservation of energy by the photon energy \( \Delta E \)). If this photon is traveling at the speed of light, it can travel a distance \( R \) such that

\[ R = c \Delta t = \frac{hc}{\Delta E} = \frac{hc}{E_\gamma} \]

where \( E_\gamma \) is the photon energy.

If the exchanged particle is not a photon but has mass \( m \), its minimum energy is its rest mass \( mc^2 \), so

\[ \Delta t \leq \frac{\hbar}{mc^2} \]

and the range \( R \) of the interaction is

\[ R \leq \frac{\hbar}{mc} \]

The exchange particles are the graviton for the gravitational force, the pion for the strong interaction between nucleons, the photon for the electromagnetic force and the \( W^\pm \) and \( Z \) bosons for the weak interaction. For an exchange particle of zero mass (the photon), the range of the force is essentially infinite. In the case of the strong interaction between nucleons, the range of the force is less than 1.4 fm, so \( m_{\text{exchange}} \geq 140 \text{ MeV/c}^2 \). In the case of the weak interaction, the exchange particles, the \( W^\pm \) and \( Z \) bosons have masses \( m \approx 90 \text{ GeV/c}^2 \), so \( R \approx 10^{-3} \text{ fm} \).
When dealing with atoms and molecules and their interactions, one is dealing primarily with the electromagnetic interaction, which is well known. In principle, the problems of atomic and molecular structure are thus soluble, albeit sometimes with a great deal of mathematical complexity. For the nuclear or strong interaction that is not the case. While we know much about nucleons and their interactions, there are some features of the nuclear force that are poorly understood even today. Since the nucleon is a composite particle, it is not surprising that the interaction between nucleons is complicated. Nonetheless, an exploration of some of the features of the nuclear force will greatly aid us in understanding nuclear phenomena.

5.2 Characteristics of the Strong Force.

As discussed earlier, the range of the nuclear force $R$ is thought to be short with $R \leq 1.4 \text{ fm}$. What evidence do we have for this? The fact that the strong force plays no role in atomic or molecular structure restricts its range to less than the nuclear radius. In our discussion of the semi-empirical binding energy equation, we showed that nuclear forces “saturate” and that nucleons only interact with their nearest neighbor. Thus the range of the nucleon-nucleon interaction must be of the order of the size of a nucleon, \textit{i.e.}, a few femtometers ($10^{-15} \text{ m}$).

We know that the nuclear force is strongly attractive, binding nucleons together to form a densely packed nucleus. Experiments involving the scattering of high energy particles from nuclei have shown the nuclear force has a repulsive core. What we mean by this statement is that below some value of the separation between nucleons ($\sim 0.5 \text{ fm}$), the nuclear force becomes repulsive instead of
attractive. (This feature, due to the quark substructure of the nucleon, prevents the nucleus from collapsing on itself).

The simplest bound nuclear system, the deuteron, consists of a neutron and a proton. The deuteron is known to have a quadrupole moment, 0.00286 barns, which tells us that the deuteron is not perfectly spherical and that the force between two nucleons is not spherically symmetric. Formally, we say the force between two nucleons has two components, a spherically symmetric central force and an asymmetric tensor force that depends on the angles between the spin axis of each nucleon and the line connecting them.

The deuteron has only one bound state, a triplet angular momentum state, in which the spins of the neutron and proton are parallel, adding to make a \( J=1 \) state. The singlet \(^1S\) state in which the nucleon spins are antiparallel is unbound. Thus the nuclear force is spin dependent. Also we shall see that the nuclear force depends on the coupling of the nucleon spin and nucleon orbital angular momentum. The deuteron magnetic moment, 0.857 nm, is close to the sum of the neutron (-1.913) and proton magnetic moments (2.793). Detailed studies show a small portion (~4%) of the time, the neutron and proton are in a \(^3D\) state \((L=2, S=1, J=1)\) rather than ground state \(^3S\) configuration \((L=0, S=1, J=1)\).

Using the relationship between force and potential energy discussed earlier, we can represent the nuclear force in terms of a simple plot of the nuclear potential energy as a function of distance to the center (Figure 5-1). Since low energy particles cannot probe the interior of nucleons or the nucleus, we can usually ignore the repulsive core in most problems involving low energy nuclear structure and just
use a square well potential \( V = -V_0 \) for \( r < R \), \( V = 0 \) for \( r > R \). Occasionally the Yukawa form of the potential is used where \( V = -V_0 \exp(-r/R)/(r/R) \) or the Woods-Saxon form where \( V = V_0/(1+\exp((r-R)/a)) \). The typical values of \( R \) for these potentials are 1.5-2 fm with \( V_0 = 30-60 \) MeV. Important additional components of the nuclear force are discussed as they become important in our discussions of nuclear structure.

5.3 Charge Independence of Nuclear Forces

The nuclear force between two nucleons is **charge-independent**. By this we mean that the strong interaction between two protons or two neutrons or a neutron and a proton is the same. (Of course, there will be differing electromagnetic forces in these cases). Evidence for the charge independence of nuclear forces can be found in nucleon-nucleon scattering and in the binding energies of light mirror nuclei. (Table 5-1) (Mirror nuclei are isobars where the number of protons in one nucleus is equal to the number of neutrons in the other nucleus and vice versa). In Table 5-1, we tabulate the total nuclear binding energy of some light mirror nuclei, the difference in Coulomb energy between the nuclei, and the resulting net ‘nuclear’ binding energy. The latter quantity is remarkably similar for these mirror nuclei, supporting the idea of charge independence of nuclear forces.

**Example**
Consider the mirror nuclei $^{25}\text{Mg}$ and $^{25}\text{Al}$. What is the energy difference between their ground states? Note the “conversion” of $^{25}\text{Mg}$ into $^{25}\text{Al}$ will involve the change of one neutron into one proton. The neutron and proton have slightly different masses, of course. The extra proton will interact electromagnetically with the other twelve protons giving a second term in the energy difference.

$$\Delta E = E(A, Z+1) - E(A, Z) = \Delta E_{\text{coul}} - (m_n - m_H)c^2$$

$$\Delta E_{\text{coul}} = \frac{6}{5} \cdot \frac{e^2}{R} \cdot Z = \frac{6}{5} \cdot \frac{(1.44/1.2 \times 25^{1/3})}{12}$$

$$(m_n - m_H)c^2 = 0.782 \text{ MeV}$$

$$\Delta E = 5.910 - 0.782 = 5.128 \text{ MeV}$$

So we would expect the ground state of $^{25}\text{Al}$ to be 5.128 MeV above the ground state of $^{25}\text{Mg}$.

The observation of the masses of mirror nuclei suggests the strong or nuclear force between a neutron and a proton is the same. This equivalence leads naturally to considering the neutron and the proton as corresponding to two states of the same particle, the nucleon. (A similar situation holds for the $\pi$ meson, where the $\pi^0, \pi^+, \pi^-$ mesons show the same strong force behavior). To express this idea, we say there is a quantum number $T$ for the nucleon (or the $\pi$ meson) called the **isospin**. In analogy to spin angular momentum, we say that for the nucleon $T=1/2$ and in this hypothetical isospin space, there are two projections of $T$, $T_3 = +1/2$ (the proton) and $T_3 = -1/2$ (the neutron). (An alternate notation system refers to the isospin projection as $T_z$). For a system with isospin $T$, there are $2T+1$ members of the isospin multiplet. In a nucleus of $N$ neutrons and $Z$ protons,
\[ T_3 = \frac{Z - N}{2} \]

For even nuclei, \(0 \leq T \leq A/2\), while for odd nuclei, \(1/2 \leq T \leq A/2\).

Isospin is a useful concept in that it is conserved in processes involving the strong interaction between hadrons. The use of isospin can help us to understand the structure of nuclei and forms the basis for selection rules for nuclear reactions and nuclear decay processes. While a detailed discussion of the effects of isospin upon nuclear structure, decay and reactions is reserved for later chapters, a few simple examples will suffice to demonstrate the utility of this concept.

Consider the \(A=14\) isobars, \(^{14}\text{C}\), \(^{14}\text{N}\), and \(^{14}\text{O}\). \(^{14}\text{C}\) and \(^{14}\text{O}\) are mirror nuclei and have ground states with \(T_3 = \pm 1\). As such they must be part of an isospin triplet with \(T=1\) (\(T_3 = 0, \pm 1\)). Thus in the \(T_3 = 0\) nucleus, \(^{14}\text{N}\), there must be a state with \(T=1, T_3 = 0\) that is the analog of the \(T_3 = 0\) ground states of \(^{14}\text{C}\) and \(^{14}\text{O}\). (See problems for further details). We expect the three members of this multiplet to have approximately the same energy levels after correction for the Coulomb effect.

In heavy nuclei, the Coulomb energy shift between members of an isospin multiplet can be large due to the large number of protons in the nucleus. Thus the isobaric analog of the ground state of one member of an isospin multiplet may lie at several MeV excitation. Thus when Fox et al., (Phys. Rev. Lett. 12, 198 (1964)) were doing routine excitation function measurements for the \(^{89}\text{Y}(p, n)^{89}\text{Zr}\) reaction, which essentially converts a neutron in the target nucleus into a proton, they observed two sharp peaks in the neutron yields near \(E_p = 5\) MeV, as shown in Figure 5-2. This observation was unexpected, as the reaction was populating levels in the
$^{90}\text{Zr}$ compound nucleus at an excitation energy of $\sim 10$ MeV where the spacing between levels was small and no states were known that produced such large resonances. Angular distributions quickly showed the $(J,\pi)$ of these states were $2^-$ and $3^-$. It was pointed out that the ground and first excited state of $^{90}\text{Y}$ had $2^-$ and $3^-$ and were separated by $\sim 200$ keV. Calculations of the Coulomb energies showed these resonances to correspond to the isobaric analogs of the ground state and first excited state of $^{90}\text{Y}$. Their yields was enhanced because they represent particularly simple nuclear configurations in contrast to the normal states found at excitation energies of 10 MeV.

**Problems**

1. Make a table like Table 5-1 showing the total binding energy, the Coulomb energy, and the net nuclear binding energy for $^{14}\text{C}$, $^{14}\text{O}$ and the 3.95 MeV level of $^{14}\text{N}$.

2. If the difference in energy of the ground state of $^{14}\text{C}$ and the $T=1$ analog of the $^{14}\text{C}$ ground state in $^{14}\text{N}$ of 3.95 MeV is due to the Coulomb energy difference between the nuclei, calculate an average radius $R$ for these $A=14$ nuclei.

3. For a Yukawa nuclear potential with $V_0 = 40$ MeV and $R = 1.5$ fm, calculate the ratio between the nuclear and Coulomb potential for $r = 1, 2, 4, 8, \text{and} 16$ fm.
Figure 5-1 Schematic representation of the radial dependence of the nuclear potential energy.
Figure 5-2 Neutron yields vs. proton energy for the reactions shown.
Table 5-1

<table>
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<tr>
<th>A</th>
<th>Nucleus</th>
<th>Total Binding Energy (MeV)</th>
<th>Coulomb Energy (MeV)</th>
<th>Net nuclear binding energy (MeV)</th>
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<tr>
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