For problems 1 and 2, consider the row equivalent matrices below.

\[
\begin{pmatrix}
-3 & 6 & -3 & -18 \\
4 & -8 & 5 & 28 \\
-2 & 4 & -2 & -12
\end{pmatrix}
\sim
\begin{pmatrix}
1 & -2 & 1 & 6 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 0
\end{pmatrix}.
\]

1. (12 pts) Use the above information to write down the solution set to the system \(Ax = b\) if

\[
A = \begin{pmatrix}
-3 & 6 & -3 \\
4 & -8 & 5 \\
-2 & 4 & -2
\end{pmatrix}
\text{ and } b = \begin{pmatrix}
-18 \\
28 \\
-12
\end{pmatrix}.
\]

2. (8 pts) For \(A\) as in problem 1, find two different column vectors \(u\) and \(v\) such that \(Au = Av\).

What does this tell you about whether or not the linear transformation \(T_A : \mathbb{R}^3 \to \mathbb{R}^3\) is one-to-one?

For problems 3 and 4, consider the row equivalent matrices below.

\[
\begin{pmatrix}
-3 & 6 & -3 & -18 \\
4 & -8 & 5 & 28 \\
-2 & 4 & -2 & -5
\end{pmatrix}
\sim
\begin{pmatrix}
1 & -2 & 1 & 6 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

3. (12 pts) Use the above information to write down the solution set to the system \(Ax = c\) if

\[
A = \begin{pmatrix}
-3 & 6 & -3 \\
4 & -8 & 5 \\
-2 & 4 & -2
\end{pmatrix}
\text{ and } c = \begin{pmatrix}
-18 \\
28 \\
5
\end{pmatrix}.
\]

4. (8 pts) For \(A\) as in problem 3, find a vector \(w\) such that \(Ax = w\) has \underline{no solution}.

What does this tell you about whether or not the linear transformation \(T_A : \mathbb{R}^3 \to \mathbb{R}^3\) is onto?

(More problems on other side of paper)
For problems 5 and 6, use the following information.
A matrix $M$ is obtained from a matrix $P$ by performing a sequence of three row operations. The row operations are the following:
First, add twice the first row to the third.
Next, interchange rows 2 and 3.
Finally, multiply row 2 by 2.

5. (12 pts) Find elementary matrices $E_1$, $E_2$, and $E_3$ such that $E_3E_2E_1P = M$. Note the order of $E_1$, $E_2$, and $E_3$.

6. (12 pts) If $M = \begin{pmatrix} -3 & 6 & -3 \\ 0 & 1 & 5 \\ 0 & 0 & -2 \end{pmatrix}$, what is the determinant of $P$?
Do not compute the entries of the matrix $P$.

7. (12 pts) A linear transformation $T$ from $\mathbb{R}^3$ to $\mathbb{R}^3$ satisfies the following:

\[
T(e_1) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \quad T(e_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad T(e_3) = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}
\]

What is the standard matrix $[T]$ associated with $T$?
Is $[T]$ invertible? Why or why not?

8. (12 pts) Write down the standard matrix of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that is described by first doing a counterclockwise rotation through an angle $\pi/4$, then doing a reflection through the $y$ axis.

9. (12 pts) If $A$ and $B$ are invertible $n$ by $n$ matrices, is $((AB)^T)^{-1}$ equal to $(A^{-1})^T (B^{-1})^T$ or is it equal to $(B^{-1})^T (A^{-1})^T$.
Justify your answer.