Chapter 3 - Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$

Skim the material in chapter 3 to see if there are any concepts that you are unfamiliar with. Most of this material is covered in multi-variable calculus.

We will generalize some of the material in Chapter 4.

Section 3.1 introduces vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$ both from a geometric point of view and from an algebraic, or coordinate point of view. The concepts of vector equality, the sum and difference of vectors, the 0 vector and scalar multiples of vectors are explained.

Section 3.2 considers the norms of vectors and vector arithmetic.

**Def.** $\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$ if $\vec{u} = (u_1, u_2, u_3)$.

$\|k \vec{u}\| = k \|\vec{u}\|$

Section 3.3 introduces dot products, both from a geometric point of view and from a coordinate point of view.

**Def.** $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos(\theta)$

$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

**Theorem 3.3.1**

$\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$

$\vec{u} \cdot \vec{u}$ can be used to determine whether $\theta$ is acute, obtuse or right.
Note: In $\mathbb{R}^2$, a vector perpendicular to the line $ax + by + c = 0$ is the vector $(a, b)$. Why?

Theorem 3.3.2 Properties of dot products

Theorem 3.3.3

$$\text{proj}_{\vec{a}} \vec{u} = \frac{\vec{u} \cdot \vec{a}}{|\vec{a}|} \cdot \frac{\vec{a}}{|\vec{a}|}$$

Distance from a point to a line

Section 3.4 introduces cross products and their properties

Section 3.5 introduces lines and planes in $\mathbb{R}^3$. 