2.2 - Determinants

Read 2.2 and try odd numbered problems.

We will take the following as our definition of determinants instead of the definition in Section 2.1

**Def:** If \( A \) is an \( n \times n \) matrix, let \( [A]_{ij} \) denote the \( n-1 \times n-1 \) matrix obtained by deleting the \( i^{th} \) row and \( j^{th} \) column of \( A \).

**Def:** The determinant of a \( 2 \times 2 \) matrix \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \) is 

\[ ad - bc \]

and is denoted \( |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \).

**Theorem 2.2.1** If \( A \) has a row or column of zeroes, then \( |A| = 0 \).

Also, \( |A| = |A^T| \)

**Proof:**

**Def.** An upper triangular matrix is an \( n \times n \) matrix with zeroes below the main diagonal. A lower triangular matrix is an \( n \times n \) matrix with zeroes above the main diagonal. A diagonal matrix is an \( n \times n \) matrix with zeroes off the main diagonal. All of these matrices are called triangular.
Theorem 2.2.2
The determinant of a triangular matrix is the product of the entries on the main diagonal.

Proof:

Examples:

Theorem 2.2.3 Effect of Elementary Row Operations
Let $B$ be obtained from $A$ by multiplying a row by a constant $k$. Then $|B| = k|A|$. Let $C$ be obtained from $A$ by interchanging two rows. Then $|B| = -|A|$. Let $D$ be obtained from $A$ by adding a multiple of one row to another. Then $|D| = |A|$.

Proof:

Theorem 2.2.4: (Determinants of elementary matrices $E$)
If $E$ is obtained from $I$ by interchanging two rows, $|E| = -1$. If $E$ is obtained from $I$ by adding a multiple of one row to another, $|E| = 1$. If $E$ is obtained from $I$ by multiplying a row by $k$, $|E| = k$.

Proof:

Note: the above results can be used to compute determinants by using row operations.

Examples: