1. What is a sequence?

2. Discuss convergence of a geometric sequence.

3. Define a sequence which has a limit, is convergent, has a convergent subsequence, is divergent, is bounded, is Cauchy, is monotone. Draw Venn diagrams which show which of the sets of sequences are subsets of which sets in the universe of “all sequences”.

4. Give an example (if possible) of
   - a sequence convergent to 3,
   - a sequence with three convergent subsequences,
   - a sequence bounded by the numbers $-1$ and $2.5$,
   - a sequence which is NOT convergent,
   - a divergent sequence,
   - an unbounded sequence,
   - a Cauchy sequence,
   - a Cauchy sequence which is not convergent,
   - a monotone sequence,
   - a non-monotone sequence which is convergent,
   - a sequence which is monotone and bounded,
   - a sequence which is monotone and unbounded,
   - a sequence which is convergent and unbounded,
   - a sequence which is bounded and divergent,
   - a sequence which is bounded and divergent to $\infty$.

5. Formulate the Bolzano-Weierstrass Theorem, sketch its proof, give example of an application.

6. Assume $\lim_{n \to \infty} x_n = a$ and $\lim_{n \to \infty} y_n = b$. What can you say about the sequence $u_n = x_n + y_n$, $z_n = \pi x_n$, $w_n = x_n y_n$? Prove it. If $\forall \ n : \ x_n \geq y_n$ then what can you say about $a$ and $b$? What if we actually know that $\forall \ n : \ x_n > y_n$?

7. Assume that, given sequences $x_n$ and $y_n$ which are each convergent to the same number $a \in \mathbb{R}$, that there is a sequence $r_n$ such that $x_n \leq r_n \leq y_n$. What can you say about $\lim_{n \to \infty} r_n$? Prove it. Give example of an application.