Mth 614 Fall 2003
Assignment 1, Nov 12
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Due - Dec 5, 2003

Problem 1. Let E be a metrizable topological vector space. If $(x_n)_{n\geq 1}$ is a sequence in E then x_n converges to 0 if and only if there exits an increasing sequence of positive real numbers $(t_n)_{n\geq 1}$ such that $\{\, t_n x_n \;\mid\; n \geq 1 \,\}$ is bounded and such that $\lim_{n\to\infty} t_n = \infty$.

If $x_n \to 0$ we can choose t_n such that $t_n x_n \to 0$.

Hint: You will need to make use of the existence of a countable base of neighborhoods of the origin.

Problem 2. Let E be a seminormed space and let F be a Banach space. Then the space $\operatorname{Hom}(E, F)$ of continuous linear maps $T: E \to F$ is a Banach space relative to the operator norm

$$\|T\| = \sup_{\|x\|\leq 1} \|T(x)\| \,.$$

In particular E' is a Banach space.
