

Bent E. Petersen	petersen@math.orst.edu
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Use this page as cover sheet stapled onto your submission, or do all your work on this page if there is sufficient room.

**Name:**

**Problem 1.** Let  $\gamma$  be a closed rectifiable curve in  $\mathbb{C}$ . Let  $w \in \mathbb{C}$ ,  $w \notin \text{traj}(\gamma)$ . Suppose we make some rough estimates of the integral  $\int_{\gamma} \frac{dz}{z-w}$  and we find

$$\left| \int_{\gamma} \frac{dz}{z-w} - 18i \right| < 5.$$

Compute the winding number  $\text{Ind}_{\gamma}(w)$  of  $\gamma$  about  $w$ .

**Problem 2.** Let  $f(x + iy) = x^3 + iy^3$  and let  $\gamma(t) = t + it^2$  for  $0 \leq t \leq 1$ . Compute

$$\int_{\gamma} f(z) dz.$$

**Problem 3.** Let  $\beta: [a, b] \rightarrow \mathbb{C}$  be a rectifiable curve. Compute the contour integral

$$\int_{\beta} e^z dz.$$

(Note  $\beta$  need not be closed.)

**Problem 4.** Let  $\beta: [a, b] \rightarrow \mathbb{C}$  be a rectifiable curve. Compute the contour integral

$$\int_{\beta} \sin z dz.$$

(Note  $\beta$  need not be closed.)

**Problem 5.** Let  $\beta: [a, b] \rightarrow \mathbb{C}$  be given by  $\beta(t) = e^{it}$  for  $0 \leq t \leq 1$  so  $\beta$  is a parametrization of the unit circle. We know

$$\int_{\beta} \frac{1}{z} dz = 2\pi i.$$

Compute each of the following integrals

$$\int_{\beta} \frac{1}{|z|} dz, \quad \int_{\beta} \frac{1}{z} |dz|, \quad \int_{\beta} \frac{1}{|z|} |dz|.$$

**Problem 6.** Compute

$$\int_{\beta} \Re(z) dz$$

where  $\beta$  is the unit circle parametrized as in the previous problem.