

Problems 1–4 require a fair amount of calculation. If you mess up the first part of your calculation the next part will drive you crazy! Here is a bit of help.

Note there was a typographical error in the original version of Problem Set 1 – the value of k_2 got switched between problems 3 and 4. It's fixed now, and also below.

For problems 1–3 the coefficient matrix is

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ -3 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \end{bmatrix}$$

The characteristic polynomial of this matrix is

$$z^4 + \frac{13}{6}z^2 + \frac{5}{6} = \left(z^2 + \frac{5}{3}\right) \left(z^2 + \frac{1}{2}\right).$$

The eigenvectors are

$$\begin{bmatrix} -3 \\ 1 \\ -2\sqrt{15}i \\ \sqrt{15}i \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ \sqrt{2}i \\ 3\sqrt{2}i \end{bmatrix}$$

and their conjugates.

For problem 4 the coefficient matrix is

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ -11 & 9 & 0 & 0 \\ 9 & -21 & 0 & 0 \end{bmatrix}$$

The characteristic polynomial of this matrix is

$$z^4 + \frac{25}{2}z^2 + 25 = (z^2 + 10) \left(z^2 + \frac{5}{2}\right).$$
