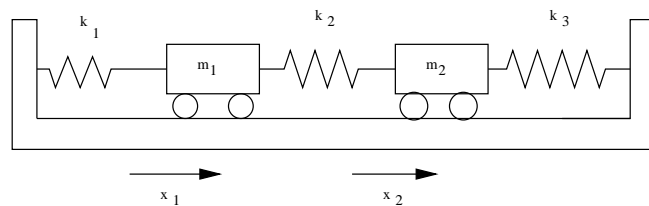


You may discuss the assigned problems with other people to get ideas (not solutions). The goal, after all is to learn some mathematics, and discussion is an effective way to do so. However, you should do your own write-up and it should be clear, brief and tidy.

Note: In the original version of this assignment I switched the values of k_2 between problems 3 and 4. This error has been corrected below.

Staple this cover sheet onto your submission.



The image above represent two wagons with frictionless wheels joined by an idealized Hooke spring and attached to a solid structure also by idealized Hooke springs. The x_1 and x_2 represent the displacement of the wagons from equilibrium.

The motion of this system is described by two linked second order ordinary differential equations.

$$m_1 \frac{d^2 x_1}{dt^2} = -(k_1 + k_2)x_1 + k_2 x_2$$

$$m_2 \frac{d^2 x_2}{dt^2} = k_2 x_1 - (k_2 + k_3)x_2$$

If we introduce the variables $x_3 = m_1 \frac{dx_1}{dt}$ and $x_4 = m_2 \frac{dx_2}{dt}$ we obtain the system of differential equations $\frac{dx}{dt} = Ax$ where

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{m_1} & 0 \\ 0 & 0 & 0 & \frac{1}{m_2} \\ -k_1 - k_2 & k_2 & 0 & 0 \\ k_2 & -k_2 - k_3 & 0 & 0 \end{bmatrix}$$

Problem 1. Let $m_1 = 2, m_2 = 3, k_1 = 2, k_2 = 1,$ and $k_3 = 1$. Solve the system above with initial condition $x_1 = 1, x_2 = 2, x_3 = 0$ and $x_4 = 0$. The solution you obtain is referred to as *push-push*. Plot x_1 and x_2 on the same graph to get a feel for the solution.

Problem 2. Let $m_1 = 2, m_2 = 3, k_1 = 2, k_2 = 1,$ and $k_3 = 1$. Solve the system above with initial condition $x_1 = -3, x_2 = 1, x_3 = 0$ and $x_4 = 0$. The solution you obtain is referred to as *push-pull*. Plot x_1 and x_2 on the same graph to get a feel for the solution.

Problem 3. Let $m_1 = 2, m_2 = 3, k_1 = 2, k_2 = 1,$ and $k_3 = 1$. Solve the system above with initial condition $x_1 = 2, x_2 = 1, x_3 = 0$ and $x_4 = 0$. The solution you obtain is not periodic. Explain why this is the generic situation. [k_2 corrected.]

Problem 4. Let $m_1 = 2, m_2 = 3, k_1 = 2, k_2 = 9,$ and $k_3 = 12$. Solve the system above with any initial conditions. Explain why you get a periodic (but generally not sinusoidal) solution. [k_2 corrected.]

Here is a plot of x_1 and x_2 for the last problem. You can clearly see the periodicity.

