Mth 480 Systems of ODE
Bent E. Petersen 20060505
Assignment 4

NAME ____________________

1 Problem. Due May 29.

You may discuss the assigned problems with other people to get ideas (not solutions). The goal, after all is to learn some mathematics, and discussion is an effective way to do so. However, you should do your own write-up and it should be clear, brief and tidy.

Staple this cover sheet onto your submission.

Recall the system of ordinary differential equations

$$\frac{dx}{dt} = A(t)x$$

where the $n \times n$ matrix $A(t)$ is continuous and periodic in t with period $T > 0$ has a fundamental solution matrix

$$\Phi(t) = P(t)\mathrm{e}^{-tR}$$

where $P(t)$ is periodic in t with period T and where R is a constant matrix. This Floquet decomposition need not be real however. We can obtain a real decomposition if we allow $P(t)$ to have period $2T$.

Thus if

$$A(t) = \begin{bmatrix} -2\cos^2(t) & -1-\sin(2t) & 0 \\ 1-\sin(2t) & -2\sin^2(t) & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

then we have the Floquet decomposition with

$$P(t) = \begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad R = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

(See text page 227 for this example.) Note that $A(t)$ is actually periodic with period π but $P(t)$ is periodic with period 2π.

Problem 7. For the example above find a Floquet decomposition where $P(t)$ is periodic with period π. (Note $P(t)$ and R will not be real.)