Mth 480 Systems of ODE
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Assignment 3

NAME

2 Problems. Due April 28.

You may discuss the assigned problems with other people to get ideas (not solutions). The goal, after all is to learn some mathematics, and discussion is an effective way to do so. However, you should do your own write-up and it should be clear, brief and tidy.

Staple this cover sheet onto your submission.

We can solve a few special variable coefficient systems. For example, consider the Cauchy-Euler system

$$t\frac{dx}{dt} = Ax$$

where A is a constant $n \times n$ matrix. If we define $y(s) = x(e^s)$ then

$$\frac{dy}{ds} = Ay.$$

Now solve this system and then set $x(t) = y(\log t)$.

Problem 5. Find the fundamental solution matrix $X(t)$ of the system

$$t\frac{dx}{dt} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} x$$

such that $X(1) = I$.

Consider the (scalar) Cauchy-Euler equation

$$at^2x'' + btx' + cx = 0$$

where $a \neq 0$, b and c are real constants. If we set

$$\begin{aligned} x_1 &= x \\ x_2 &= tx' \end{aligned}$$

we obtain the system

$$t\frac{dx}{dt} = Ax$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{c}{a} & 1-\frac{b}{a} \end{bmatrix} \quad \text{and } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Problem 6. Convert $t^2x'' + tx' + x = 0$ to a system as above. Then solve the system and finally find $x = x_1$.