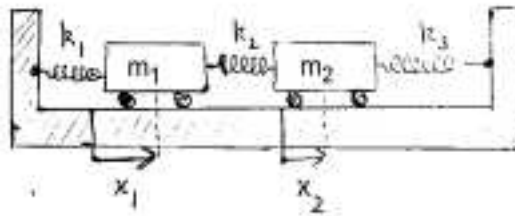


NAME _____

1 Problem. Due April 26.

You may discuss the assigned problems with other people to get ideas (not solutions). The goal, after all is to learn some mathematics, and discussion is an effective way to do so. However, you should do your own write-up and it should be clear, brief and tidy.

Staple this cover sheet onto your submission.



Problem 4. Consider two frictionless carts with masses m_1 and m_2 joined by Hooke springs with spring constants k_1 , k_2 and k_3 to each other and to a solid frame as indicated in the illustration above. Then the equations of motion are given by

$$m_1 \frac{d^2 x_1}{dt^2} = -k_1 x_1 + k_2 (x_2 - x_1)$$

$$m_2 \frac{d^2 x_2}{dt^2} = -k_2 (x_2 - x_1) - k_3 x_2.$$

If we introduce the momenta $x_3 = m_1 \frac{dx_1}{dt}$ and $x_4 = m_2 \frac{dx_2}{dt}$ we obtain the system of first order equations

$$\frac{dx}{dt} = Ax \quad \text{where} \quad A = \begin{bmatrix} 0 & 0 & 1/m_1 & 0 \\ 0 & 0 & 0 & 1/m_2 \\ -(k_1 + k_2) & k_2 & 0 & 0 \\ k_2 & -(k_2 + k_3) & 0 & 0 \end{bmatrix}.$$

Consider the system where $m_1 = 1, m_2 = 2$ and $k_1 = 3, k_2 = 2, k_3 = 1$.

1. Compute e^{tA} to find the general solution of the equations of motion.
2. Find the solution with initial conditions $x_1(0) = 1, x_2(0) = 2, x_3(0) = 0, x_4(0) = 0$. Describe this solution.
3. Find the solution with initial conditions $x_1(0) = 4, x_2(0) = -1, x_3(0) = 0, x_4(0) = 0$. Describe this solution.
4. Find the solution with initial conditions $x_1(0) = 1, x_2(0) = 1, x_3(0) = 0, x_4(0) = 0$. Plot this solution for $t = 0$ to $t = 50$. Comment on what you see.