Mth 480 Systems of ODE
Bent E. Petersen 20060406
Assignment 1

NAME ______________________________

3 Problems. Due April 14.

You may discuss the assigned problems with other people to get ideas (not solutions). The goal, after all is to learn some mathematics, and discussion is an effective way to do so. However, you should do your own write-up and it should be clear, brief and tidy.

Staple this cover sheet onto your submission.

Problem 1. Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

Compute the operator norm (that is, the 2,2-norm) of A by first computing the eigenvalues of $A^T A$.

Problem 2. Let B be a lower triangular $n \times n$ matrix. Show e^B (defined by the Taylor series of the exponential) is lower triangular with diagonal entries the exponentials of the diagonal entries of B. Conclude for each eigenvalue λ of B, we have e^λ is an eigenvalue of e^B, with the same multiplicity (apart from a small difficulty caused by the complex exponential not being one-to-one). Conclude

$$\det\left(\mathrm{e}^A\right) = \mathrm{e}^{\mathrm{tr}(A)}$$

for any $n \times n$ matrix A. Here $\det$ is the determinant and tr is the trace. You may find it useful to know that any square matrix is similar to a lower triangular matrix.

Problem 3. Let

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}.$$

Compute the eigenvalues and corresponding eigenvectors of A. Find an invertible matrix S such that $S^{-1}AS$ is diagonal. Use your results to find the general solution of the system of ordinary differential equation

$$\frac{dx}{dt} = Ax.$$