Mth 355 Exam | Name: ID:

Bent Petersen `355f2001_exam.tex` Dec 3, 2001 Time: 110 minutes.

This test is open book. You may use any books or notes that you wish to use. You may use a calculator.

Problem 1. *(20 points if correct, 0 points if wrong).* Let X be a set of cardinality 7 and let Y be a set of cardinality 5. The number of ways of partitioning X into 5 nonempty subsets is 140. Determine the number of epimorphisms (surjections, onto-maps) $f: X \to Y$.

A.) 120 **B.)** 140
C.) $5,040$ **D.)** $16,800$ **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 1.*

Problem 2. *(20 points if correct, 0 points if wrong).* Let A be the power set of the empty set $\emptyset$ and let B be the power set of A. The cardinality of B is

A.) 0 **B.)** 1
C.) 2 **D.)** ∞ **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 2.*

Problem 3. *(20 points if correct, 0 points if wrong).* In a class of 19 good students 6 students will be assigned a grade of A, 11 students will be assigned a grade of B and the remainder will be assigned a grade of C. How many ways are there to assign the grades?

A.) $6,859$ **B.)** $27,132$
C.) $2,116,296$ **D.)** $1,162,261,467$ **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 3.*

Problem 4. *(20 points if correct, 0 points if wrong).* An urn contains a large number of red, white and blue marbles. How many ways are there of selecting 9 marbles from the urn if the number of red marbles must be odd, but not 3, and no more than 7, and if the number of white marbles must be even, but not 4, and no more than 6? **Hint:** Consider the generating function $g(x) = (x + x^5 + x^7)(1 + x^2 + x^6)(1 + x^2 + x^4 + x^6 + x^8)$.

A.) 7 **B.)** 8
C.) 9 **D.)** 10 **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 4.*

Problem 5. *(20 points if correct, 0 points if wrong).* The generating function for a certain sequence $(a_n)_{n\geq 0}$ is

$$g(x) = \frac{1}{(1-x^2)(1-x^4)}.$$

Find a_6.

A.) 2 **B.)** 3
C.) 4 **D.)** 5 **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 5.*

Problem 6. *(20 points if correct, 0 points if wrong).* Solve the recurrence relation

$$a_n = 2\,a_{n-1} + 2^n, \quad a_0 = 1.$$

Then compute a_{20}.

A.) 20 **B.)** 1,048,576
C.) 20,971,520 **D.)** 22,020,096 **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 6.*

Problem 7. *(20 points if correct, 0 points if wrong).* Solve the recurrence relation

$$a_n = 4\,a_{n-2} + 2^n, \quad a_0 = 1, a_1 = 0.$$

Then compute a_{10}.

A.) 1025 **B.)** 1363
C.) 1365 **D.)** 1385 **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 7.*

Problem 8. *(20 points if correct, 0 points if wrong).* The adjacency matrix of a simple graph is (always)

A.) singular **B.)** invertible
C.) symmetric **D.)** nonsquare **E.)** None of the foregoing.

☐ ←*Letter corresponding to your answer to problem 8.*

Additional test policies for this class are provided on my web page `http://www.onid.orst.edu/~peterseb`.

Use this space for scratch work.

Please do not write in the boxes to the right. They are for your grades.
Do not be concerned if there are more boxes than problems.

						Letter Grade ☐ *This test only* ☐ *Cummulative*				
1	2	3	4	5	6	7	8	9	10	Total

Note: There are 8 problems for a total of 160 points.