Mth 351 Assignment 3 Winter 2003 | Due: Mar 14, 2003

Bent Petersen 351w2003_asg03.tex

For a certain function f on the interval $[0,3]$ the (compound) trapezoidal rule with 2^k subintervals yields the following estimates for the integral $\int_0^3 f(x)\,\mathrm{d}x$,

k	0	1	2	3	4	5	6	7	8
	1.6015118	1.6516437	2.6206689	2.5851766	2.5969237	2.5998804	2.6006201	2.6008051	2.6008513

Problem 1. Given the data above use Richardson extrapolation to find the (compound) Simpson's rule approximations with 2^7 and with 2^8 subintervals to the integral $\int_0^3 f(x)\,\mathrm{d}x$.

Problem 2. If we apply the Richardson extrapolation once again to the Simpson's rule approximations from the previous problem what estimate of the integral do we obtain?

In the problems below f denotes a continuous function on the interval $[0,1]$.

Problem 3. Develop a quadrature method of the form

$$Q(f) = A\,f(0) + B\,f(\frac{1}{5}) + C\,f(\frac{3}{5}) + D\,f(1)$$

to estimate the integral of $f(x)$ on the interval $[0,1]$. Choose the constants A, B, C and D such that your method is exact for polynomials of degree ≤ 3.

Problem 4. Apply the method of the previous problem to $f(x) = \exp(x)$. What is the error?

Problem 5. Develop a quadrature method of the form

$$Q(f) = A\,f(0) + B\,f(\frac{3}{4}) + C\,f(1)$$

to estimate the integral of $f(x)$ on the interval $[0,1]$. Choose the constants A, B, and C such that your method is exact for $p(x)\mathrm{e}^x$ where $p(x)$ is any polynomial of degree ≤ 2.

Problem 6. Apply the method of the previous problem to $f(x) = x^2$. What is the error?

Turn in a reasonable amount of work, but don't overdo it.

Rules. *You may talk to anyone and get help wherever you can for any assignment, but at some point you must write up your work by yourself.*