

Let

$$\begin{aligned}f(x, y) &= x^2 - 3y^2 + 2xy^2 - 4x + 7y - 2 \\g(x, y) &= 2x^3 + 3y^2 + 6xy - 5x + 2y - 2.\end{aligned}$$

Then the nonlinear system of equations

$$\begin{aligned}f(x, y) &= 0 \\g(x, y) &= 0\end{aligned}$$

can be shown to have 6 solutions,

$$\begin{aligned}(-2.175, -0.8586), & (-1.243, 1.747), (-0.4339, 0.01092), \\(0.4218, 0.6196), & (2.864, -3.173), (2.438, -4.429).\end{aligned}$$

Suppose we apply Newton's iterative method for approximating roots three times, that is, make 3 steps, starting with the initial guess $(0, 0)$.

Problem 1. Do we appear to have convergence towards one of the roots above? Which one?

Problem 2. Compute the 2-norm of the residual vector of the last estimate found above.

You will need to do quite a bit of calculation to solve this problem and you will need to do it at fairly high precision. A calculator, or even better, a computer is definitely needed.

Turn in a reasonable amount of work, but don't overdo it.

Rules. *You may talk to anyone and get help wherever you can for any assignment, but at some point you must write up your work by yourself.*