Mth 351 Assignment 1 Winter 2003 | Due: Jan 28, 2003

Bent Petersen 351w2003_asg01.tex

A real number x_a is said to have m significant decimal digits as an approximation to the real number $x_t \neq 0$ provided that

$$\left| \frac{x_t - x_a}{x_t} \right| \le 5 \times 10^{-m-1}.$$

The best value of m is clearly given by

$$m = \left\lfloor - \log_{10} \left(2 \left| \frac{x_t - x_a}{x_t} \right| \right) \right\rfloor .$$

Here $\lfloor \quad \rfloor$ indicates the *greatest integer function* otherwise known as *floor*.

Problem 1. The fractions $\frac{22}{7}$, $\frac{333}{106}$ and $\frac{355}{113}$ are well-known approximations to π. Use the formula above to find the the number of significant decimal digits in each. Does your understanding of significant digits agree with your calculated result?

Problem 2. Use the central symmetric formula

$$f''(a) = \frac{f(a-h) - 2f(a) + f(a+h)}{h^2} + \mathcal{O}(h^2)$$

to estimate the second derivative of $\exp(x)$ at the origin. Compute the error for step sizes $h = 0.1, 0.05, 0.025$ and 0.0125. Do your results agree with the order 2 claim for this method?

Rules. *You may talk to anyone and get help wherever you can for any assignment, but at some point you must write up your work by yourself.*