Mth 351 Test 1 | Summer 2005 | (This test requires you use the provided scantron)

Bent Petersen `351u2005-test1.tex` Test date: July 25 2005 Time: 70 minutes

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

- *This test is a multiple-choice test. Be sure you put your name on the scantron.*
- *You must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.*
- *You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator or a modest graphics calculator available for use on this test. Calculators and other equipment may not be shared.*
- *You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).*
- *There are 10 multiple-choice problems worth 10 points each.*

Be sure to enter all required information on the scantron.

Section Number: 001
Form Number: 001

Important Notes:

- Note that $\log(x)$ means the *natural logarithm* of x, sometimes denoted by $\ln(x)$. The logarithm with base 10 will be denoted by $\log_{10}(x)$, the logarithm with base 2 will be denoted by $\log_2(x)$, and so on.
- Return only the scantron. You may keep the test (and your note sheet).
- Make certain your calculator is set to radian mode.

Problem 1. Suppose the Taylor polynomial of degree 4, with center at the origin, for $f(x)$ is given by

$$P(x) = 1 + x - x^2 + \frac{1}{3}x^3 - \frac{1}{20}x^4$$

and suppose

$$\left| f^{(5)}(x) \right| \le \frac{13}{100}, \quad \text{for } |x| \le 1.$$

Find an upper bound for the absolute error in $P(0.5)$ as an approximation to $f(0.5)$. Choose the best error bound from the list below.

A.) 3.39×10^{-7} **B.)** 2.86×10^{-6}
C.) 3.39×10^{-5} **D.)** 2.86×10^{-4} **E.)** 1.08×10^{-3}

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

Problem 2. Let $f(x) = x^2 + 2x + \sin(x)$. Let our initial guess for a root be $x_0 = -2.0$ and apply Newton's method twice to obtain a new estimate x_2 for a root. Choose the value closest to x_2.

A.) -2.376 **B.)** -2.318
C.) -2.317 **D.)** -2.313 **E.)** -2.293

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

Problem 3. Let $P(x)$ be the interpolation polynomial of degree ≤ 3 through the points $(0,2)$, $(1,2)$, $(2,1)$ and $(4,3)$. Compute $P(3)$.

A.) $3/4$ **B.)** 1
C.) $5/4$ **D.)** $7/24$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

Problem 4. Let $P(x)$ be the interpolation polynomial of the previous problem. Let f be any "smooth" function interpolating the points of the previous problem. If $\left| f^{(4)}(x) \right| \leq 3/8$ for $0 \leq x \leq 4$ then the absolute error in $P(3)$, when viewed as an approximation to $f(3)$, is bounded by: (choose the best bound)

A.) 0.0157 **B.)** 0.0375
C.) 0.0500 **D.)** 0.0938 **E.)** 0.1012

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

Problem 5. Find the interpolation polynomial of degree at most 3 for the points $(0,0)$, $(1,1)$, $(2,-1)$, $(3,2)$.

A.) $y = \frac{3}{2}x^3 - 6x^2 + \frac{11}{2}x$ **B.)** $y = \frac{7}{6}x^3 - 5x^2 + \frac{29}{6}x$
C.) $y = \frac{4}{3}x^3 - \frac{11}{2}x^2 + \frac{31}{6}x$ **D.)** $y = \frac{7}{3}x^3 - \frac{17}{2}x^2 + \frac{43}{6}x$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

Problem 6. Find the least squares fit of the form $y = a + bx^2$ for the data points $(0,1)$, $(1,1)$, $(2,5)$.

A.) $y = 1 + 2x^2$ **B.)** $y = \frac{1}{2} + \frac{14}{13}x^2$
C.) $y = \frac{1}{3} + 2x^2$ **D.)** $y = \frac{7}{13} + \frac{14}{13}x^2$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

Problem 7. Consider a (very unrealistic) computer with floating point numbers with a mantissa of length only 9 bits. Assume the mantissa is not packed and in addition that chopping is used to fit numbers to the limited storage. Assuming no problem in storing the exponent, which of the following numbers is closests to the error made in storing 3.7?

A.) 4.688×10^{-3} **B.)** 1.445×10^{-2}
C.) 1.347×10^{-3} **D.)** 1.113×10^{-2} **E.)** 2.229×10^{-4}

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

Problem 8. Let $f(x) = x^2 + 2x + \sin(x)$. Let our initial guesses for a root be $x_0 = -2.0$ and $x_1 = -2.1$. Apply the secant method twice to obtain a new estimate x_3 for a root. Choose the value closest to x_3.

A.) -2.376 **B.)** -2.318
C.) -2.317 **D.)** -2.313 **E.)** -2.293

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

Problem 9. Given $f(0.9) = 2.4$, $f(1.0) = 2.3$ and $f(1.1) = 2.9$ estimate $f''(1.0)$.

A.) 7 **B.)** 70
C.) 3.5 **D.)** 35 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 9).*

Problem 10. We know Euler's number is given by

$$\mathrm{e} = \sum_{k=0}^{\infty} \frac{1}{k!}.$$

(Note the sum starts at $k = 0$,) What is the smallest *number of terms* we should add to guarantee an error no larger than 7.5×10^{-5}?

A.) 8 **B.)** 9
C.) 11 **D.)** 13 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 10).*

$$f'(a) = \frac{f(a+h) - f(a-h)}{2h} + \mathcal{O}\left(h^2\right)$$

$$f''(a) = \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} + \mathcal{O}\left(h^2\right)$$

Use the backs of the test pages for scratch work.