

Bent Petersen 351u2002_assignment_01.tex

Let $0 \leq c \leq 4$ and let

$$f(x) = cx(1 - x).$$

Then f maps the interval $[0, 1]$ into itself. If $c < 1$ then f is a contraction and so has a unique fixed point (namely 0). Moreover for any initial x_0 the fixed point iteration

$$x_{n+1} = f(x_n)$$

converges fairly rapidly to the fixed point. For $c \geq 1$ we no longer have a contraction mapping and the behavior of the fixed point iterates changes significantly with increasing c .

Problem 1. When $1 < c \leq 4$ there are 2 fixed points. What are they (in terms of c).

Problem 2. Use initial values $1/4, 1/2, 3/4$, compute a few iterates and comment on their apparent behavior. Do this for $c = 2, c = 3$ and $c = 3.9$.

You can, with patience, do the calculations with a calculator, or use Matlab, Maple or even a spreadsheet. If you elect to use a spreadsheet you may find useful the (ancient) book

William J. Orvis, *1-2-3 for Scientists and Engineers*, Sybex, San Francisco, 1987.

You may also find the Maple worksheet `351u2002_logistic.mws` (or `351u2002_logistic.pdf` if you do not have Maple handy) on my webpage helpful to see what is going on.

Rules. You may talk to anyone and get help wherever you can for any assignment, but at some point you must write up your work by yourself. If you write some code and ask me to help to debug your code, you will get a well-deserved large frown. Life's too short to debug my own code, let alone others' code.