Here is a list of common formulæ for numeric differentiation. I have not included any approximations of order one since they are not very useful. Be careful if you decide to use any of these formulæ – numeric differentiation is inherently unstable.

$$
\begin{aligned}
* \quad f^{(1)}(a) &= \frac{-f(a-h)+f(a+h)}{2h} + \mathcal{O}\big(h^2\big) \\
f^{(1)}(a) &= \frac{-3\,f(a)+4\,f(a+h)-f(a+2h)}{2h} + \mathcal{O}\big(h^2\big) \\
f^{(1)}(a) &= \frac{-2\,f(a-h)-3\,f(a)+6\,f(a+h)-f(a+2h)}{6h} + \mathcal{O}\big(h^3\big) \\
f^{(1)}(a) &= \frac{-11\,f(a)+18\,f(a+h)-9\,f(a+2h)+2\,f(a+3h)}{6h} + \mathcal{O}\big(h^3\big) \\
* \quad f^{(1)}(a) &= \frac{f(a-2h)-8\,f(a-h)+8\,f(a+h)-f(a+2h)}{12h} + \mathcal{O}\big(h^4\big) \\
f^{(1)}(a) &= \frac{-3\,f(a-h)-10\,f(a)+18\,f(a+h)-6\,f(a+2h)+f(a+3h)}{12h} + \mathcal{O}\big(h^4\big) \\
f^{(1)}(a) &= \frac{-25\,f(a)+48\,f(a+h)-36\,f(a+2h)+16\,f(a+3h)-3\,f(a+4h)}{12h} + \mathcal{O}\big(h^4\big)
\end{aligned}
$$

$$
\begin{aligned}
* \quad f^{(2)}(a) &= \frac{f(a-h)-2\,f(a)+f(a+h)}{h^2} + \mathcal{O}\big(h^2\big) \\
f^{(2)}(a) &= \frac{2\,f(a)-5\,f(a+h)+4\,f(a+2h)-f(a+3h)}{h^2} + \mathcal{O}\big(h^2\big) \\
* \quad f^{(2)}(a) &= \frac{-f(a-2h)+16\,f(a-h)-30\,f(a)+16\,f(a+h)-f(a+2h)}{12h^2} + \mathcal{O}\big(h^4\big) \\
f^{(2)}(a) &= \frac{11\,f(a-h)-20\,f(a)+6\,f(a+h)+4\,f(a+2h)-f(a+3h)}{12h^2} + \mathcal{O}\big(h^3\big) \\
f^{(2)}(a) &= \frac{35\,f(a)-104\,f(a+h)+114\,f(a+2h)-56\,f(a+3h)+11\,f(a+4h)}{12h^2} + \mathcal{O}\big(h^3\big)
\end{aligned}
$$

$$
\begin{aligned}
* \quad f^{(3)}(a) &= \frac{-f(a-2h)+2\,f(a-h)-2\,f(a+h)+f(a+2h)}{2h^3} + \mathcal{O}\big(h^2\big) \\
f^{(3)}(a) &= \frac{-3\,f(a-h)+10\,f(a)-12\,f(a+h)+6\,f(a+2h)-f(a+3h)}{2h^3} + \mathcal{O}\big(h^2\big) \\
f^{(3)}(a) &= \frac{-5\,f(a)+18\,f(a+h)-24\,f(a+2h)+14\,f(a+3h)-3\,f(a+4h)}{2h^3} + \mathcal{O}\big(h^2\big)
\end{aligned}
$$

$$
* \quad f^{(4)}(a) = \frac{f(a-2h)-4\,f(a-h)+6\,f(a)-4\,f(a+h)+f(a+2h)}{h^4} + \mathcal{O}\big(h^2\big)
$$

In general expected the order of the estimate is the number of points minus the order of the derivative. In the central (*) formulæ for even order derivatives, the order of the estimate is one higher than expected. In the central (*) formulæ for odd order derivatives, function evaluation at the central point drops out.

Note in each non-central formula we can interchange the number of forward and backward steps by replacing h by $-h$. These non-central formulæ are useful near the ends of a table of data, where there are not enough data points for a central (*) formula.

*Bent Petersen File ref: 351s2001-numeric-derivatives.tex