Linear Algebra – Mth 341

Archive – Spring 1995 Files

Feb 27, 2001

This archive contains two assignments from Mth 341 Spring 1995. The tests and other files from this quarter of Mth 341 have been lost. If I find some more of them I will add them to this archive.

Contents

1 Assignment 1

Let

$$A := \begin{pmatrix} 0 & 4 & 8 & 12 & 0 & 4 & -4 & 8 & 9 & 12 \\ 2 & 2 & 0 & 8 & 11 & -1 & 2 & 3 & 8 & 6 \\ 1 & 1 & 0 & 4 & 6 & -1 & 2 & -1 & 4 & 2 \\ 1 & 1 & 0 & 4 & 5 & 0 & 0 & 2 & 3 & 1 \\ 1 & 1 & 0 & 4 & 5 & 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 4 & 5 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 & 0 & 1 & -1 & 2 & 2 & 3 \end{pmatrix}$$

Problem 1. Identify the six pivot columns and four free variables for the matrix A.

Problem 2. Denote the i-th column of the matrix A by $\mathbf{a_i}$. Let the set C be the set of all pivot columns of A. Show that $\text{span}(\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_{10}})$ $=\text{span}(C)$.

For each column corresponding to a free variable, do the following: If it's the i-th column, form the vector $\mathbf{v_i}$ as follows:

Set the i-th free variable equal to 1 and all of the other free variables equal to 0. Then use the homogeneous system $A\mathbf{v} = \mathbf{0}$ to solve for the other components of $\mathbf{v_i}$.

Problem 3. Perform the above procedure to obtain the four $\mathbf{v_i}$ vectors for A.

Problem 4. Prove that the vectors in Problem 3 are linearly independent.

Let $T : \mathbf{R}^{10} \longrightarrow \mathbf{R}^7$ be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$.

Problem 5. Show that every linear combination of the four $\mathbf{v_i}$ is in $\ker(T)$, the kernel of T.

Problem 6. Show that every element of $\ker(T)$ is a linear combination of these four $\mathbf{v_i}$.

2 Assignment 2

Problem 7. For the matrix A below, use Maple to find the eigenvalues of A. Also, find the eigenspace corresponding to each eigenvalue.

$$A := \begin{pmatrix} 7 & -1 & 1 \\ 7 & -1 & -5 \\ 6 & -6 & 0 \end{pmatrix}.$$

Note: Part of this question is for you to figure out how Maple can be used. Please append your Maple output to the work you hand in.

Problem 8. Changing even one entry to floating point puts Maple into floating point calculations. In the above matrix A, change each entry which is 7 to 7.0, and answer the same questions as in problem 1 above.

Problem 9. Contrast the outputs from problems 1 (7) and 2 (8). In your description, be sure to describe the output differences and also try to give an explanation of why those differences might have occurred.

Instructions for the three remaining problems. Let V be the vector space of 3×3 matrices with real entries, with usual matrix addition and usual matrix scalar multiplication. Let $\mathbf{e_{1,1}}, \mathbf{e_{1,2}}, \ldots \mathbf{e_{3,3}}$ be the elements of the standard basis, where I'm thinking of $\mathbf{e_{i,j}}$ as the 3×3 matrix in which the only nonzero entry is in the i-th row and j-th column, and it's equal to 1.

Problem 10. Consider the function $S : V \longrightarrow V$ defined by $S(\mathbf{v}) = \mathbf{v}^T$. Prove that the function S is a linear transformation from V to V.

Problem 11. Find the matrix of the transformation S, using the standard basis for V.

Problem 12. Decide whether or not the following six matrices are linearly independent in V. Show all your work. (By the way, it's okay to use Maple on any calculations, as long as you clearly explain what you are doing.)

$$A_1 := \begin{pmatrix} 7 & -1 & 1 \\ 7 & -1 & -5 \\ 6 & 3 & 4 \end{pmatrix}; A_2 := \begin{pmatrix} 1 & 5 & 6 \\ 7 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}; A_3 := \begin{pmatrix} 2 & 3 & 1 \\ 0 & 0 & 3 \\ 4 & -1 & 2 \end{pmatrix};$$

$$A_4 := \begin{pmatrix} 2 & -1 & 0 \\ 0 & 0 & 2 \\ 2 & 0 & 1 \end{pmatrix}; A_5 := \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & -1 \\ 1 & 2 & 0 \end{pmatrix}; A_6 := \begin{pmatrix} 0 & 0 & 5 \\ 6 & 1 & -1 \\ 2 & 2 & 0 \end{pmatrix}.$$

3 Contact Information

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