Mth 311 Test 1 F97 | Name:

Bent Petersen `311f97q1.tex` November 21, 1997 Time: 50 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one note–sheet prepared in advance. You must put your name on your note–sheet. Your note–sheet may not be larger than 8.5 by 11 inches (22 by 28 cm). You may write on both sides of your note–sheet.*
- *You may not use any books nor additional note sheets.*
- *You may use a calculator, but it is difficult to imagine what you will use it for. Calculators and other equipment may not be shared.*
- *Write your solutions neatly. Use the backs of the examination sheets for scratch work. There is a blank (mostly) page at the end of the test which may also be used for rough work.*
- *Partial credit will be assigned only for clear, correct, legible work. Remember the emphasis is on clear correct mathematical arguments. If you tell me that*

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

converges, you are right, but you will not get much of a grade unless you also tell me convincingly how you deduced it converges.

Problem 1. *(30 points).*

Part (A): Give a careful statement of the Bolzano–Weierstrass theorem.

Part (B): Give an example of a sequence with no convergent subsequence.

Problem 2. *(30 points).* Prove if a monotone sequence has a convergent subsequence, then it converges. (You may assume the sequence is monotone increasing.) Be sure to give a careful complete proof.

Problem 3. *(30 points).* Consider the series

$$\sum_{n=1}^{\infty} 2^{(-1)^n - n}$$

What conclusion can you draw from the root test? What conclusion can you draw form the ratio test? (Please use the version of the ratio test that we did in class, if possible, rather than the one in the text.)

Problem 4. *(30 points).* Investigate the convergence or divergence of the following series:

Part (A): $$\sum_{n=1}^{\infty} \frac{1}{n^{\log n}}$$

Part (B): $$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n}$$

Part (C): $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

Problem 5. *(30 points).* Note multiplication by 2 gives a one–to–one correspondence of $\mathbb{N}$ with a proper subset of $\mathbb{N}$. From this fact is easy to conclude that each countable set may be put in one–to–one correspondence with a proper subset of itself. (Note — a one–to–one correspondence is a function which is one–to–one (injective) and onto (surjective).)

Part (A): Give an argument (not too formal) that each infinite set contains a countable subset.

Part (B): Use part (A) and the assertions made above to show that any infinite set may be put in one–to–one correspondence with a proper subset of itself.

This page is (mostly) blank. Use it for overflow (neatly labelled) and scratch work, but do not remove it.

Please do not write in the boxes to the right. They are for your grades.

1	2	3	4	5	6	7	8	9	10	Total

Note: There are 5 problems for a total of 150 points.