# Mth 306 Midterm | Name: ID:

Bent Petersen `306w2005-test.tex` Feb 9, 2005 Time: 50 minutes.

**Instructions:** $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to enter all required information on the scantron and on this test.

Section Number: 001
Form Number: 001

- *This test is multiple-choice and workout. You must turn in both the test and the scantron.*
- *For the multiple-choice problems you must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.*
- *For the workout problems you must show your work in reasonable detail on the test. Partial credit is allocated only for clear and relevant work.*
- *You may use one $8.5 \times 11$ inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator available for use on this test. Calculators and other equipment may not be shared.*
- *You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).*
- *There are 9 multiple-choice (8 points each) and 2 work-out (20 points each) problems.*
- *Note that $\log(x)$ means the* natural logarithm *of $x$.*

**Important Note:** If you are taking this test in the Mathematics Learning Center you will not need a scantron. Just be sure to write the letters corresponding to your answers in the boxes provided below.

**Problem 1.** The infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\left(\log(n)\right)^2}$

**A.)** converges by ratio test **B.)** diverges by ratio test
**C.)** converges by integral test **D.)** diverges by integral test **E.)** None of the above.

[ ] ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

**Problem 2.** The infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\log(n)}$

**A.)** converges by ratio test **B.)** diverges by ratio test
**C.)** converges by integral test **D.)** diverges by integral test **E.)** None of the above.

[ ] ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

**Problem 3.** The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n(n+2)} = \sum_{n=1}^{\infty}\left(\frac{2}{n} - \frac{2}{n+2}\right)$

**A.)** diverges **B.)** converges with sum $2$
**C.)** converges with sum $3$ **D.)** converges with sum $4$ **E.)** None of the above.

[ ] ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

**Problem 4.** If we substitute $u = \sin(x)$ in the series $\sum_{n=0}^{\infty}(-1)^n \frac{\sin^{n+1}(x)}{2^n}$ we obtain a convergent geometric series. Therefore the sum of this series is

**A.)** $\frac{2\sin(x)}{2-\sin(x)}$ **B.)** $\frac{2\sin(x)}{2+\sin(x)}$

**C.)** $\frac{1}{2-\sin(x)}$ **D.)** $\frac{1}{2+\sin(x)}$ **E.)** None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

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**Problem 5.** Suppose $\lim_{n\to\infty} n^2 a_n = 3$. What can we say about the series $\sum_{n=1}^{\infty} a_n$?

**A.)** divergent alternating series **B.)** convergent alternating series

**C.)** diverges by limit comparison test **D.)** converges by limit comparison test **E.)** None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

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**Problem 6.** Use the root test or the ratio test to investigate the convergence of the series $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{n^n}$

**A.)** converges absolutely, limiting root or ratio $< 1$ **B.)** converges, but only conditionally

**C.)** diverges, limiting root or ratio $> 1$ **D.)** test fails, limiting root or ratio $= 1$ **E.)** None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

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**Problem 7.** Use the root test or the ratio test to investigate the convergence of the series $\sum_{n=1}^{\infty}(-1)^n \left(\frac{5n+4}{4n+5}\right)^n$

**A.)** converges absolutely, limiting root or ratio $< 1$ **B.)** converges, but only conditionally

**C.)** diverges, limiting root or ratio $> 1$ **D.)** test fails, limiting root or ratio $= 1$ **E.)** None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

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**Problem 8.** Use the root test or the ratio test to investigate the convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^5}{5^n}$

**A.)** converges absolutely, limiting root or ratio $< 1$ **B.)** converges, but only conditionally

**C.)** diverges, limiting root or ratio $> 1$ **D.)** test fails, limiting root or ratio $= 1$ **E.)** None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

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**Problem 9.** Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!}{n^{2n}} x^n$.

**A.)** 0 **B.)** 1

**C.)** e **D.)** $\infty$ **E.)** None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 9).*

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**Problem 10.** Find the sum of the series $\sum_{n=1}^{\infty} nx^n$ for $|\,x\,| < 1$. **Hint:** Differentiate the geometric series term-by-term.

**Problem 11.** One can show if $|\,x\,| < 1$ then

$$\log\left(\frac{1+x}{1-x}\right) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$$

(James Gregory 1668). For the corresponding Taylor polynomial approximation we have

$$\log\left(\frac{1+x}{1-x}\right) = 2\sum_{k=0}^{n} \frac{x^{2k+1}}{2k+1} + R(x)$$

where $R(x)$ is the appropriate remainder. The remainder is painful to compute using the Lagrange formula but we can estimate it by the sum of a geometric series,

$$|\,R(x)\,| \leq \frac{2}{2n+3} \sum_{k=n+1}^{\infty} |\,x\,|^{2k+1} \leq \frac{2}{2n+3}\,|\,x\,|^{2n+3} \sum_{k=0}^{\infty} |\,x\,|^{2k}.$$

If we take $x = 1/3$ we obtain an approximation to $\log(2)$. If we take $n = 2$ (that is, the Taylor polynomial of degree 5) what upper bound does the error estimate above yield?

Use this page and the backs of all the pages for scratch work.