Mth 306 Exam

(Make sure your name and ID number are on the scantron.)

Bent Petersen `306s2005-exam.tex` June 9 2005 Time: 110 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to enter all required information on the scantron.

Section Number: 001
Form Number: 001

- *This test is a multiple-choice test. Be sure you put your name on the scantron.*
- *You must mark your answer on the provided scantron. Fill in the appropriate bubbles on the scantron very carefully.*
- *You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator available for use on this test. Calculators and other equipment may not be shared.*
- *You may use a simple graphics calculator but not a laptop computer nor any device capable of extensive symbolic manipulation (other than your own brain).*
- *There are 18 multiple-choice problems worth 8 points each.*

Important Notes:

- Note that $\log(x)$ means the *natural logarithm* of x, sometimes denoted by $\ln(x)$. The logarithm with base 10 will be denoted by $\log_{10}(x)$, the logarithm with base 2 will be denoted by $\log_2(x)$, and so on.
- Return only the scantron. You may keep the test (and your note sheet).

Problem 1. Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ k & k & 2 \\ 1 & 3 & 2 \end{bmatrix}.$$

A.) 0 **B.)** k
C.) $4k - 2$ **D.)** $k^2 - 2$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

Problem 2. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ then $AB - BA = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ where $e =$

A.) 0 **B.)** 3
C.) 5 **D.)** 7 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

Problem 3. The augmented matrix of a system of linear equations has the row reduced echelon form

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The system has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** only the trivial solution. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

Problem 4. The augmented matrix of a system of linear equations has the row reduced echelon form

$$\begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The system has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** only the trivial solution. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

Problem 5. The augmented matrix of a system of linear equations is

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -2 & 3 \\ 1 & 4 & 4 & 2 \end{bmatrix}.$$

The system has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** only the trivial solution. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

Problem 6. The augmented matrix of a system of linear equations is

$$\begin{bmatrix} 2 & -3 & 3 & 3 \\ 1 & -3 & 1 & -1 \\ 1 & 0 & 2 & 3 \end{bmatrix}.$$

The system has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** only the trivial solution. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

Problem 7. The augmented matrix of a system of linear equations is

$$\begin{bmatrix} 2 & -3 & 3 & 3 \\ 1 & -3 & 1 & -1 \\ 1 & 0 & 2 & 4 \end{bmatrix}.$$

The system has

A.) no solutions. **B.)** exactly one solution.
C.) infinitely many solutions. **D.)** only the trivial solution. **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

Problem 8. Let

$$A = \begin{bmatrix} 1 & 4 & 2 \\ k & 4 & k \\ 3 & 2 & 1 \end{bmatrix}.$$

For what values of k does the system of linear equations $A\vec{x} = \vec{0}$ have infinitely many solutions?

A.) $k = 1$ **B.)** $k = 2$
C.) $k = 3$ **D.)** all k **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

Problem 9. Let

$$A = \begin{bmatrix} 1 & 0 & k \\ 0 & 1 & 0 \\ k & 0 & 1 \end{bmatrix}$$

where k is any real number. Then the largest eigenvalue of A is

A.) $1 - k$ **B.)** $1 - |\,k\,|$
C.) $1 + k$ **D.)** $1 + |\,k\,|$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 9).*

Problem 10. The matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

has real eigenvalues. One of its eigenvalues is -1. Find a corresponding eigenvector (note: ^{T} indicares the transpose.)

A.) $[0, -1, 1]^T$ **B.)** $[1, 0, 0]^T$
C.) $[0, 1, 1]^T$ **D.)** $[2, 1, -1]^T$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 10).*

Problem 11. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

The largest eigenvalue is

A.) 2 **B.)** 5

C.) 6 **D.)** 8 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 11).*

Problem 12. The vector $[8, 4, 1, -3]^T$ is an eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

Find the corresponding eigenvalue.

A.) 2 **B.)** 5

C.) 6 **D.)** 8 **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 12).*

Problem 13. The columns of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & -1 \\ 3 & 5 & 1 \end{bmatrix}$$

A.) form a basis of $\mathbb{R}^3$ **B.)** span (or generate) $\mathbb{R}^3$

C.) span a proper subspace of $\mathbb{R}^3$ **D.)** are linearly independent **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 13).*

Problem 14. The columns of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

A.) form a basis of $\mathbb{R}^3$ **B.)** do not span (or generate) $\mathbb{R}^3$

C.) span $\mathbb{R}^3$ but do not form a basis **D.)** are not linearly independent **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 14).*

Problem 15. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n^{2n}}\, x^n$$

A.) e^2 **B.)** 4
C.) $\mathrm{e}^2/4$ **D.)** $4/\mathrm{e}^2$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 15).*

Problem 16. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} n^3 3^n\, x^n$$

A.) $1/3$ **B.)** 1
C.) 3 **D.)** ∞ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 16).*

Problem 17. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} n!\, x^n$$

A.) 0 **B.)** 1
C.) 3 **D.)** ∞ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 17).*

Problem 18. Find the sum of the power series

$$\sum_{n=1}^{\infty} n^2\, x^n$$

A.) $2/(1-x)^3$ **B.)** $x/(1-x)^3$
C.) $(1+x^2)/(1-x)^3$ **D.)** $x(1+x)/(1-x)^3$ **E.)** None of the foregoing.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 18).*

Use this page and the backs of all the pages for scratch work.

Have a great Summer!