NEWTON's law of cooling states if an object at temperature T is in contact with an object at temperature A then, ignoring all other effects, the rate of change of the temperature T is proportional to the difference in the temperatures. Thus

$$\frac{dT}{dt} = -k(T - A), \qquad \text{or} \qquad \frac{dT}{dt} + kT = kA,$$

where k is a constant.

Problem 0121 – 1. A cup of hot coffee initially at temperature 125.0° F is brought into a room of temperature A (the ambient). The coffee begins to cool down and, of course the room warms up a bit. However, the heat capacity of the room is so large compared to the cup of coffee, that we may assume A remains constant. After 2 minutes the coffee is observed to have the temperature 107.0° K. Another minute later the coffee is observed to have the temperature 100.6° F. Deduce the temperature of the room.

The next problem deals with a root cellar, an insulated box, a building heated by the sun, or perhaps a wine cellar. We assume that the ambient temperature varies sinusoidally about a certain mean, and we wish to compute the interior temperature.

Problem 0121 – 2. Assume we have a sealed root cellar and that the ambient temperature varies sinusoidally about a mean A_0 with the amplitude of the fluctuation given by A_1. Then

$$A(t) = A_0 + A_1 \sin(\omega t)$$

Compute the resulting temperature T in the root cellar (in terms of A_0, A_1, k and a parameter determined by initial conditions). Identify the steady state component of $T(t)$ and compute the phase shift and the attenuation factor in the case $A_0 = 18°$ C, $A_1 = 4°$ C, $\omega = 5\pi$, $k = 3$. You may assume $T(0) = 0$ if you wish – it makes no difference.

The graph below show the ambient $A(t)$ and the response $T(t)$. You can use it as a rough check on your calculations.