Mth 256 Test | Winter 2006 | Name:

Bent Petersen `256w2006-test.tex`
Date: Wednesday, Feb 15, 2006 Time: 50 min.

- If a scantron is provided with this test then fill in your ID information on the scantron now. Also enter your name on this test in the space provided above. Do not fold, staple or tear, etc., the scantron. Return both the entire test and the scantron (separately). If a scantron is not provided with this test, then ignore the scantron instructions, here and below.

- This test has some multiple-choice problems and perhaps some work-out problem(s). Fill in the answers to the multiple-choice problems in the boxes below and on the scantron (if one is provided). The answer(s) to the work-out problem(s) must be accompanied by "work" and must be written on this test.

- You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet. Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes. You may not use any books, notebooks, additional note sheets nor note cards. Be sure to put your name on your note sheet, but do not turn it in.

- You may use a simple scientific calculator or a modest graphics calculator on this test and you are expected to have one available. An overly elaborate calculator, laptop, handheld or notebook computer, or any device capable of extensive symbolic manipulation (other than your own brain) will not be allowed. Calculators and other equipment may not be shared.

- There are 7 multiple-choice problems worth 10 points each. and 1 workout problem(s) worth 25 points each.

Multiple-choice problems: 7 problems

Problem 1. Solve the initial value problem

$$\frac{dy}{dt} = \left(1 + y^2\right) t, \quad y(0) = 1.$$

A.) $y(t) = \tan(t^2/2) + 1$

B.) $y(t) = \tan(t) + 1$

C.) $y(t) = \tan(t^2/2 + \pi/4)$

D.) $\log(y^2 + 1) = t^2/2$

E.) None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 1).*

Problem 2. Solve the initial value problem

$$\frac{dy}{dt} = \frac{t^2 + y^2}{ty}, \quad y(1) = -1.$$

A.) $y = \sqrt{1 + 2\log(t)}\, t$

B.) $y = -\sqrt{1 + 2\log(t)}\, t$

C.) $y = -\sqrt{1 + 2\log(t)}$

D.) $y = -\sqrt{1 + \log(t)}\, t$

E.) None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 2).*

Problem 3. Solve the initial value problem

$$\frac{dy}{dt} + y = y^3, \quad y(0) = 2$$

A.) $y = \frac{2}{\sqrt{4-3\mathrm{e}^{2t}}}$

B.) $y = \frac{2}{\sqrt{2-\mathrm{e}^{2t}}}$

C.) $y = 2\,\mathrm{e}^{-t}$

D.) $y = \frac{4}{\sqrt{4-3\mathrm{e}^{2t}}}$

E.) None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 3).*

Problem 4. A thermometer initially reading $68°$ F is brought into a hot room with temperature $A = 92°$ F. The temperature reading T displayed by the thermometer satisfies Newton's law

$$\frac{dT}{dt} = -k\,(T - A)$$

where k is a positive constant. After 1 minute the thermometer displays $82°$ F. What temperature will it display after an additional 2 minutes. Choose the closest number from the list below.

A.) 86.3

B.) 87.8

C.) 89.3

D.) 90.2

E.) 91.1

☐ ←*Mark answer here and on the scantron* *(Problem 4).*

Problem 5. Solve the initial value problem

$$\frac{d^2y}{dt^2}\,\frac{dy}{dt} = 3, \quad y(1) = 1, \quad y'(1) = 1.$$

Then compute $y(5)$.

A.) $\frac{107}{9}$

B.) $\frac{117}{9}$

C.) $\frac{125}{9}$

D.) $\frac{133}{9}$

E.) None of the foregoing.

☐ *←Mark answer here and on the scantron* *(Problem 5).*

Problem 6. The differential equation

$$(12y^2 + 9xy) + (18xy + 6x^2)\frac{dy}{dx} = 0$$

has an integrating factor of the form $\mu = x^p\, y^q$. Find p and q.

A.) $p = 1, q = 1$

B.) $p = 2, q = 1$

C.) $p = 1, q = 2$

D.) $p = 2, q = 2$

E.) None of the foregoing.

☐ *←Mark answer here and on the scantron* *(Problem 6).*

Problem 7. Find a 2-parameter family of solutions $y(t)$ of

$$y'' + y' - 6y = 0.$$

Then determine the parameters so that $y(0) = 0$ and $y'(0) = 5$. Then compute $y(\log(2))$.

A.) 1

B.) 2

C.) 3

D.) 4

E.) None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 7).*

Workout problem: 1 problem

Problem 8. A 100 gallon tank initially contains 40 gal brine of concentration 2 oz/gal salt. Brine of concentration 1 oz/gal flows into the tank at 5 gal/min and the well-mixed solution is drawn off at 3 gal/min. Find the concentration of salt in the brine in the tank at the very moment of overflow.

Use the space below and the backs of the test pages for scratch work.