Laplace transform tables extracted from the final exam.

Mth 256 Exam | **Name:** **ID:**

Bent Petersen `256w2004-exam.tex` March 15, 2004 Time: 110 minutes.

Instructions: $\Longrightarrow$
If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- *You may use one* 8.5×11 *inch note sheet prepared in advance. You may write on both sides of your note sheet.*
- *Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes.*
- *You may not use any books, notebooks, additional note sheets nor note cards.*
- *You are expected to have a simple scientific calculator available for use on this test. Calculators and other equipment may not be shared.*
- *For multiple-choice problems place the letter corresponding to your answer in the box provided.*
- *Note that* $\log(x)$ *means the* natural logarithm *of* x.

Don't get nervous! The Laplace transform tables below are much more extensive than what you will need on this test.

Some Laplace exchange formulæ

If $\mathcal{L}\{f(t)\}(s) = F(s)$ then

$$\mathcal{L}\left\{e^{at}f(t)\right\}(s) = F(s-a)$$

$$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s) = \int_s^\infty F(r)\ \mathrm{d}r \quad \left(\text{if } \frac{f(t)}{t} \text{ integrable at } 0\right)$$

$$\mathcal{L}\left\{\int_0^t f(r)\ \mathrm{d}r\right\}(s) = \frac{F(s)}{s}$$

$$\mathcal{L}\left\{\frac{df}{dt}\right\}(s) = sF(s) - f(0) \quad (\text{if } f \text{ cont. on } [0,\infty))$$

$$\mathcal{L}\left\{\frac{d^2f}{dt^2}\right\}(s) = s^2F(s) - sf(0) - f'(0) \quad (\text{if } f,\ f' \text{ cont. on } [0,\infty))$$

$$\mathcal{L}\{u(t-a)f(t-a)\}(s) = e^{-as}F(s) \quad (u = \text{ unit step or Heaviside function})$$

$$\mathcal{L}\{f(at)\}(s) = \frac{1}{a}F\left(\frac{s}{a}\right).$$

If $\mathcal{L}\{f(t)\}(s) = F(s)$ and $\mathcal{L}\{g(t)\}(s) = G(s)$ then $\mathcal{L}\{(f*g)(t)\}(s) = F(s)G(s)$ where $f*g$ is defined by $(f*g)(t) = \int_0^t f(t-r)g(r)\ \mathrm{d}r$.

Some Laplace transforms

$$\mathcal{L}\left\{1\right\}(s) = \frac{1}{s}$$

$$\mathcal{L}\left\{e^{at}\right\}(s) = \frac{1}{s-a}$$

$$\mathcal{L}\left\{t^n\right\}(s) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\left\{\cos\omega t\right\}(s) = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}\left\{\sin\omega t\right\}(s) = \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}\left\{e^{at}\cos\omega t\right\}(s) = \frac{s-a}{(s-a)^2+\omega^2}$$

$$\mathcal{L}\left\{e^{at}\sin\omega t\right\}(s) = \frac{\omega}{(s-a)^2+\omega^2}$$

$$\mathcal{L}\left\{e^{at}\cosh\nu t\right\}(s) = \frac{s-a}{(s-a)^2-\nu^2}$$

$$\mathcal{L}\left\{e^{at}\sinh\nu t\right\}(s) = \frac{\nu}{(s-a)^2-\nu^2}$$

$$\mathcal{L}\left\{\sqrt{t}\right\}(s) = \frac{\sqrt{\pi}}{2s^{3/2}}$$

$$\mathcal{L}\left\{t^n e^{at}\right\}(s) = \frac{n!}{(s-a)^{n+1}}$$

$$\mathcal{L}\left\{u(t-a)\right\}(s) = \frac{e^{-as}}{s} \quad (u = \text{ unit step or Heaviside function})$$

$$\mathcal{L}\left\{\delta(t-a)\right\}(s) = e^{-as} \quad (\delta = \text{ Dirac delta})$$

If f is periodic with period $T > 0$ then $\mathcal{L}\left\{f(t)\right\} = \dfrac{\int_0^T e^{-st} f(t)\ dt}{1-e^{-sT}}$.

$$\mathcal{L}\left\{1+\sum_{k=1}^{\infty}(-1)^k u(t-k)\right\}(s) = \frac{1}{s\,(1+e^{-s})}$$

$$\mathcal{L}\left\{|\sin(t)|\right\}(s) = \frac{\coth\left(\frac{\pi s}{2}\right)}{1+s^2}$$