

You may use a notesheet, prepared in advance, and no larger than  $8.5 \times 11$  inches in size. You are expected to have a scientific calculator, and you may use it. Please note  $\log(x)$  means the natural logarithm of  $x$  on this test.

This test is multiple-choice. Work carefully. Try to avoid errors and try to avoid being misled by the offered answers.

There are 8 problems for a total of 160 points.

**Problem 1.** (20 points if correct, 0 points if wrong). Find the general solution of the differential equation

$$x \frac{dy}{dx} = (1 + x^3)(1 + y^2).$$

- A.)  $y = \tan\left(x + \frac{x^3}{3} + C\right)$       B.)  $y = \tan\left(\frac{x^4}{4} + \log(x) + C\right)$   
 C.)  $y = \log\left(\frac{x^3}{3} + \log(x) + C\right)$       D.)  $y = \tan(\arctan(x) + C)$       E.) None of the foregoing.

←Letter corresponding to your answer to problem 1.

**Problem 2.** (20 points if correct, 0 points if wrong). Solve the initial value problem

$$\frac{dy}{dx} = e^{-x-y}, \quad y(0) = \log 3.$$

- A.)  $y = \log(1 + e^x)$       B.)  $y = \log(3e^x - 1) - x$   
 C.)  $y = \log(4e^x - 1) - x$       D.)  $y = \log(2 + x)$       E.) None of the foregoing.

←Letter corresponding to your answer to problem 2.

**Problem 3.** (20 points if correct, 0 points if wrong). Solve the differential equation

$$x \frac{dy}{dx} + y = 3x^2.$$

- A.)  $y = x^2 + C$       B.)  $y = x^2 + C/x$   
 C.)  $y = x^3 + C/x$       D.)  $y = x^3 + C$       E.) None of the foregoing.

←Letter corresponding to your answer to problem 3.

**Problem 4.** (20 points if correct, 0 points if wrong). Solve the exact ordinary differential equation

$$2xy + y^2 - y + 3x^2 + 1 + (x^2 + 2xy - x - 4y + 2) \frac{dy}{dx} = 0$$

- A.)  $x^2y + xy^2 - xy + x^3 - 2y^2 + x + 2y = C$   
 B.)  $x^2y + 2xy^2 - xy + x^3 - 2y^2 + x + 2y = C$   
 C.)  $x^2y + xy^2 - 2xy + x^3 - 2y^2 + x + 2y = C$   
 D.)  $x^2y + xy^2 - xy + 2x^3 - 2y^2 + x + 2y = C$   
 E.) None of the foregoing.

←Letter corresponding to your answer to problem 4.

**Problem 5.** (20 points if correct, 0 points if wrong). The ordinary differential equation

$$y(x + y + 1) + (x + 2y)\frac{dy}{dx} = 0$$

has an integrating factor depending only on  $x$ . Find such an integrating factor.

- A.)**  $x^2$     **B.)**  $\log(x)$   
**C.)**  $e^x$     **D.)**  $x$     **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 5.

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**Problem 6.** (20 points if correct, 0 points if wrong). If we substitute  $y = xv$  in the differential equation

$$\frac{dy}{dx} = \frac{xy + y^2 + x^2}{x^2}$$

we obtain

- A.)**  $\frac{dv}{dx} = v^2 + 1$     **B.)**  $x\frac{dv}{dx} = v^2 + 1$   
**C.)**  $x\frac{dv}{dx} = v^2 + v + 1$     **D.)**  $x\frac{dv}{dx} = v^2 + 2v + 1$     **E.)** None of the foregoing.

←Letter corresponding to your answer to problem 6.

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**Problem 7.** (20 points if correct, 0 points if wrong). A very large tank contains 40 L brine of concentration 2 g/L salt. Brine of concentration 1.2 g/L salt flows into the tank at 4 L/min and the well-mixed solution is pumped out at 2 L/min. Assuming that the tank does not overflow what is the concentration of salt in the brine in the tank after 10 min? (Choose the closest value.)

- A.)** 1.555 g/L    **B.)** 1.448 g/L  
**C.)** 1.327 g/L    **D.)** 1.298 g/L    **E.)** 1.000 g/L

←Letter corresponding to your answer to problem 7.

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**Problem 8.** (20 points if correct, 0 points if wrong). Given the initial value problem

$$\frac{dy}{dx} = x + y^2, \quad y(0) = 1$$

estimate  $y(0.6)$  by using EULER's method with step size  $h = 0.2$ . Choose the closest number from the list below.

- A.)** 1.97560    **B.)** 2.07495  
**C.)** 2.64399    **D.)** 2.94948    **E.)** 3.11037

←Letter corresponding to your answer to problem 8.

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