

**Instructions:**  $\implies$ 

If you do not read the instructions, then how will you know what to do? Read them now.

Be sure to write your name in the space above.

- You may use one note-sheet prepared in advance. You must put your name on your note-sheet, but do not turn in your note-sheet. Your note-sheet must be letter size,  $8.5 \times 11$  inches, or A4 paper,  $21 \times 29.7$  cm, or smaller. You may write on both sides of your note-sheet.
- Note-sheets may not be shared. If you do not bring a note-sheet you will have to do without any help notes.
- You may not use any books, notebooks nor additional note-sheets.
- You may use a calculator. Calculators and other equipment may not be shared.
- For work-out problems sketch your work neatly. Highlight your answer by drawing a frame around it. Scratch out irrelevant or incorrect work so it will be clear what you are submitting as a solution. Give exact answers when possible. Simplify your answer when reasonable to do so. Partial credit will be assigned only for relevant, clear, correct, legible work. If you do not show some relevant work or explain your solution, your grade may be 0.
- For multiple-choice problems indicate your choice in the answer box provided. You need not show any work nor offer any explanations for your answer. If you need to do some work, you may do it in the space provided, if any, or on the back of the examination sheets, but your work will not be graded. **You will be graded only on the letter you select and put in the provided answer box.** Note this test does not use a scantron.

**Problem 1.** (40 points). A 150 gallon tank initially contains 50 gallons of brine of concentration 1 oz salt per gallon. Brine of concentration 2 oz salt per gallon flows into the tank at 4 gallons per minute. The well-mixed solution is pumped out at 2 gallons per minute. Find the concentration of salt in the brine in the tank at the very moment of overflow.

**Problem 2.** (40 points). Suppose a tank containing water has an outlet in the bottom. If  $h$  is the head, that is, the height of the fluid surface above the outlet, then TORRICELLI's principle implies

$$A(h) \frac{dh}{dt} = -\gamma a \sqrt{2gh}$$

where

- $A(h)$  = horizontal cross-sectional area of the tank at height  $h$
- $a$  = cross-sectional area of the outlet
- $g$  = acceleration of gravity, 32.2 ft/sec<sup>2</sup>
- $\gamma$  = BORDA's factor, about 0.6 for water.

Suppose the tank is hemispherical, curved side down, and has a height of 12 feet, so  $A(h) = \pi(24h - h^2)$ . Suppose the outlet, which has a cross-sectional area of 0.036 ft<sup>2</sup>, is suddenly opened. If the tank is initially full of water, how long does it take (in seconds) to drain completely?

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**Problem 3.** (40 points). A 55 gallon drum containing hazardous waste, and weighing  $W = 549.8$  lb, is dropped in the ocean. Its equation of motion is

$$m \frac{dv}{dt} = mg - kv - B$$

where  $B = 470.5$  lb. is the bouyancy,  $v$  is the downward component of the velocity,  $k = 0.08$  lb-sec/ft is an experimentally determined drag coefficient,  $m$  is the mass of the drum and its contents, and  $g = 32.2$  ft/sec<sup>2</sup> is the acceleration of gravity. If  $y$  is the distance below the sea surface then  $v = \frac{dy}{dt}$  and, by the chain rule,

$$v \frac{dv}{dy} = \frac{g}{W} (W - B - kv)$$

where  $W = mg$  is the weight of the drum and its contents. Assume the drum falls from rest at the surface and compute at what depth the drum hits 34 feet per second.

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**Problem 4.** (40 points). Find the general solution *in real form* for each of the following ordinary differential equations.

**Part (A)**  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 15y = 0$

**Part (B)**  $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 13y = 0$

**Part (C)**  $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 0$

**Part (D)**  $\frac{d^3y}{dt^3} - 2\frac{d^2y}{dt^2} + \frac{dy}{dt} = 0$

Additional test policies for this class are provided on my web page <http://ucs.orst.edu/~peterseb>.

Please do not write in the boxes to the right. They are for your grades.

Do not be concerned if there are more boxes than problems.

	Letter Grade <input type="checkbox"/> <i>This test only</i> <input type="checkbox"/> <i>Cummulative</i>
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1	2	3	4	5	6	7	8	9	10	Total
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**Note:** There are 4 problems for a total of 160 points.