Applied Differential Equations – Mth 256

Archive – Summer 1996 Files

Oct 10, 2000

This archive contains the tests from Mth 256 Summer 1996. The original test instructions, headers and formatting have not been preserved.

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1 Test 1

Problem 1. *(20 points).* A 400 gallon tank contains 100 gallons of brine of concentration 1.5 oz salt per gallon. Brine of concentration 2.0 oz salt per gallon flows into the tank at 6 gallons per minute. The well-mixed solution is pumped out at 4 gallons per minute. Find the concentration of the brine solution in the tank at the end of 50 minutes.

Problem 2. *(15 points).* Solve the initial value problem

$$\frac{dy}{dx} = \frac{y \log y}{x \log x}, \quad y(2) = 16.$$

Problem 3. *(15 points).* Solve the initial value problem

$$\frac{dy}{dx} = \frac{4y^3 + 4yx^2}{3y^2x + 2x^3}, \quad y(1) = 1.$$

Problem 4. *(25 points).* The temperature in Kidder 364 on a certain day is given by

$$A(t) = A_0 - A_1 \mathrm{e}^{-\mu t}$$

where A_0, A_1 and μ are positive constants. If a cup of hot coffee is brought into the room then according to Newton the temperature T of the coffee satisfies

$$\frac{dT}{dt} = -k\,(T - A)$$

where k is a constant. Since the coffee has no perceptible effect on the temperature of the room we may assume $A(t)$ is given by the formula above. Assuming $T(0) = T_0$ and $k \neq \mu$ find the temperature T of the coffee as a function of time.

Problem 5. *(15 points).* Solve the exact ordinary differential equation

$$\left(4xy^3 - 18xy + 6x\right)\,dx + \left(9y^2 + 6x^2y^2 - 9x^2\right)\,dy = 0.$$

Problem 6. *(15 points).* The ordinary differential equation

$$\left(4y - 10xy^2\right)\,dx + \left(3x - 8x^2y\right)\,dy = 0$$

has an integrating factor of the form

$$\mu = x^m y^n.$$

Find this integrating factor. Then solve the ordinary differential equation.

2 Test 2

Problem 7. *(15 points).* Given that $y_1 = x$ and $y_2 = x^2$ are solutions of the linear homogeneous ordinary differential equation $x^2y'' - 2xy' + 2y = 0$ solve the initial value problem

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0, \quad y(1) = 2,\ y'(1) = -1.$$

Problem 8. *(42 points).* Find the general solution *in real form*. The primes denote differentiation with respect to x.

(A) $y'' - 4y' = 0$

(B) $y'' - 4y = 0$

(C) $y'' + 3y' + 2y = 0$

(D) $y'' + 2y' + 2y = 0$

(E) $y''' - y = 0$

(F) $y''' + y'' - y' - y = 0$

(G) $y''' + 2y'' - y' - 2y = 0$

Problem 9. *(23 points).* Given

$$\left(\lambda^2 + 4\lambda + 5\right)^3 = \lambda^6 + 12\lambda^5 + 63\lambda^4 + 184\lambda^3 + 315\lambda^2 + 300\lambda + 125$$

solve the linear homogeneous ordinary differential equation

$$y^{(6)} + 12y^{(5)} + 63y^{(4)} + 184y^{(3)} + 315y^{(2)} + 300y^{(1)} + 125y = 0.$$

Problem 10. *(25 points).* A certain nondissipative spring is stretched 0.15 m by a force of 5 N. The spring satisfies Hooke's hypothesis over the range of loads of interest to us. Assume the acceleration of gravity is 9.8 m/sec^2. Remark: The *kilopond*, an obsolete metric unit of force, is defined as 9.80665 N. It is roughly the *weight* of 1 kg at sea level on the Earth.

(A) Find the spring constant.

(B) A mass of 2 kg is suspended from the spring and allowed to reach equilibrium. The mass is then pulled down 0.40 m and released. How much time will pass before the mass is observed to travel through the equilibrium position in an upward direction for the first time? At what speed does it pass through the equilibrium position?

3 Test 3

Problem 11. *(20 points).* Use variation of parameters to find the general solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{\mathrm{e}^{3x}}{x^2+1}.$$

Problem 12. *(20 points).* For each of the following Cauchy–Euler ordinary differential equations find the general solution in real form:

(A) $x^2y'' + 3xy' - 3y = 0$

(B) $x^2y'' + 5xy' + 13y = 0$

(C) $x^2y'' + 7xy' + 9y = 0$

(D) $x^3y''' + 6x^2y'' + 7xy' + y = 0$

Problem 13. *(15 points).* Find the Laplace transform of the solution of the initial value problem

$$3\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 17y = \mathrm{e}^{2t}\cos t, \quad y(0) = -2,\ y'(0) = 3.$$

It is not necessary to solve the initial value problem.

Problem 14. *(20 points).* Find the inverse Laplace transforms:

(A) $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s^2-1)(s+3)}\right\}$

(B) $\mathcal{L}^{-1}\left\{\dfrac{s+3}{(s^2+1)s}\right\}$

Problem 15. *(30 points).* Find the *form* of the particular solution provided by the method of undetermined coefficients (but leave the coefficients undetermined).

(A) $y'' + y' - 6y = x\mathrm{e}^{3x} - x^2\mathrm{e}^{-3x}$

(B) $y'' - 2y' + 5y = 2x\mathrm{e}^{-x}\sin(2x).$

4 Contact Information

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