Applied Differential Equations – Mth 256

Archive – Summer 1994 Files

Jan 9, 2001

This archive contains the sample problems and tests from Mth 256 Summer 1994. The original test instructions, headers and formatting have not been preserved.

Contents

1 Sample Problems Set 1

Problem 1. A cup of coffee initially at 190° F is brought into a room at 65° F. After 2 minutes the temperature of the coffee is 145° F. Predict the temperature of the coffee an additional minute later.

If T is the temperature of the coffee then according to Newton's law of cooling

$$\frac{dT}{dt} = -k\,(T - A)$$

where A is the temperature of the room. Note we may assume that A is constant since the room has much higher heat capacity than the coffee.

Problem 2. A certain radioactive substance decays to 85 % of its original mass in 36 hours. Find the half–life.

Problem 3. Substitute $w = e^y$ to solve

$$e^y \frac{dy}{dt} + e^y = e^t.$$

Problem 4. Solve the Bernoulli ordinary differential equation

$$\frac{dy}{dt} - y = t\,y^2.$$

Problem 5. Solve (carefully)

$$\frac{dy}{dt} = 2\,t\,y^2, \quad y(1) = 0.$$

Problem 6.

$$\frac{dy}{dt} = \frac{y - 4\,t}{t - y}$$

Problem 7.

$$\frac{dy}{dt} = \frac{y^2 + 2\,t\,y}{t^2}$$

Problem 8.

$$\frac{dy}{dt} = (t + y)^2$$

Problem 9.

$$\frac{dy}{dt} = \frac{t^2 + y^2}{t^2}$$

Problem 10. For what of k is $\left(x^2 + y^2\right)^k$ an integrating factor for

$$-y\,dx + x\,dy = 0?$$

Problem 11.

$$\frac{dy}{dx} = \frac{x + y}{x - y}$$

2 Sample Problems Set 2

Problem 12.

$$\frac{dy}{dt} + 2\,t\,y = 0, \quad y(0) = -2$$

Problem 13.

$$t\,\frac{dy}{dt} + y = 0, \quad y(1) = 4$$

Problem 14.

$$t\,\frac{dy}{dt} - 4\,y = 0, \quad y(2) = -8$$

Problem 15.

$$\frac{dy}{dt} + \tan(t)\,y = 0, \quad y(0) = 2$$

Problem 16.

$$\frac{dy}{dt} + \frac{y}{t} = 0, \quad y(1) = -3$$

Problem 17.

$$\frac{dy}{dt} + \frac{t}{y} = 0, \quad y(1) = -3$$

Problem 18.

$$t^2\,\frac{dy}{dt} + y = 0, \quad y(1) = 1$$

Problem 19.

$$\frac{dy}{dt} + \log(t)\,y = 0, \quad y(2) = 1.$$

Problem 20.

$$\frac{dy}{dt} = \frac{y}{t-3} + t^2$$

Problem 21.

$$\frac{dy}{dt} = \cot(t)\,y + \sin(t)$$

Problem 22.

$$\frac{dy}{dt} = \frac{y}{t} + \sin\left(t^2\right)$$

Problem 23. Find a function $y(t)$ such that

$$y(t) + \int_0^t y(s)\,ds = t^2.$$

3 Sample Problems Set 3

Problem 24. Find the general solution of each of the ordinary differential equations

$$\textbf{(A)}\quad \frac{dy}{dx}\cos x = y\,\sin x \qquad \textbf{(B)}\quad x^2\,\frac{dy}{dx} = y^2 + 3\,xy.$$

Problem 25. The differential equation

$$\left(y - xy^2\right)\,dx + \left(x + x^2y^2\right)\,dy = 0$$

has an integrating factor of the form $x^m y^n$. **(A)** Find the integrating factor. **(B)** Solve the differential equation.

Problem 26. A 50 liter tank initially contains 10 liters of brine of concentration 0.5 gram/liter salt. A brine solution containing 1 gram/liter salt runs into the tank at the rate 4 liter/min. and the well-stired solution is drained off at the rate 2 liter/min. Find the concentration of salt in the brine in the tank at the onset of overflow.

Problem 27. Assume that the acceleration of gravity is 9.8 m/sec^2 so that a 10 kg mass will weigh 98 newtons.

A 10 kg mass is suspended from a spring, stretching it by 0.7 m. The mass is started in motion by pulling it down 0.5 m and releasing it. Assume air resistance has magnitude $90\,\dfrac{dx}{dt}$ newtons where x is the downward displacement of the mass from equilibrium.

(A) Find the equation of motion of the mass and solve it using the appropriate initial values. **(B)** How many times does the mass pass through the equilibrium position after being released?

Problem 28. Find the general solution (*in real form*) for each of the following ordinary differential equations.

$$\textbf{(A)}\quad x^2\,\frac{d^2y}{dx^2} + 11\,x\,\frac{dy}{dx} + 169\,y = 0 \qquad \textbf{(B)}\quad \frac{d^2y}{dx^2} + 4\,\frac{dy}{dx} + 5\,y = 0 \qquad \textbf{(C)}\quad x^2\,\frac{d^2y}{dx^2} - 5\,x\,\frac{dy}{dx} + 9\,y = 0.$$

Problem 29. Use variation of parameters to find the general solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} - y = \frac{e^{2x}}{1 + e^{x}}.$$

Problem 30. Use the method of undetermined coefficients to find the general solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} - 6\,\frac{dy}{dx} + 25\,y = 3\,e^{3x}\,\cos(4x).$$

Problem 31. Compute the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{2\,s^2+3}{s^3+2\,s^2-3\,s}\right\}.$$

Problem 32. Consider the initial value problem

$$\frac{d^2y}{dt^2}+3\,\frac{dy}{dt}-4\,y=t\,\mathrm{e}^t,\quad y(0)=-2,\ \ y'(0)=3.$$

Find the Laplace transform of the solution to this initial value problem.

4 Test 1

Problem 33. Solve the initial value problem

$$\frac{dy}{dx}=\frac{x\,y+y}{x},\ \ y(1)=-2.$$

Problem 34. Find the general solution of the ordinary differential equation

$$\frac{dy}{dx}=\frac{x}{y}+\frac{y}{x}.$$

Problem 35. The ordinary differential equation

$$(6\,y-4\,y^2)\,dx+(9\,x-8\,x\,y)\,dy=0$$

has an integrating factor of the form

$$\mu=x^m\,y^n.$$

Find m and n and then solve the given ordinary differential equation.

Problem 36. Solve the initial value problem

$$\frac{dy}{dx}=\frac{(x+y)^2-3}{2},\ \ y(0)=0.$$

Problem 37. Given the 1–parameter family of curves

$$y^2=\alpha\,\mathrm{e}^{x^2+y^2},\ \ (\alpha\text{ is the parameter})$$

find the family of orthogonal trajectories.

Problem 38. A tank of volume 400 L initially contains 200 L of brine of concentration $\frac{1}{5}$ g/L of salt. Brine of concentration 1 g/L flows into the tank at 8 L/min. The well–mixed solution is drawn off at the rate 6 L/min. Find the concentration of salt in the tank at the moment that it overflows.

Problem 39. Consider a cup of coffee in a room of temperature $A = 70°$ F. The cup is sitting on a small heating pad which is supposed to keep the coffee warm. If T is the temperature of the coffee then

$$\frac{dT}{dt} = -k\,(T - A) + U$$

where U is a constant depending on the heater and the cup and k is a constant depending on the cup. Initially the temperature of the coffee is 183° F, 5 minutes later the temperature is 155° F, and an additional 5 minutes later the temperature is 135° F.

Find the temperature T as a function of time. Compute

$$\lim_{t\to\infty} T(t).$$

Problem 40. The ordinary differential equation

$$\left(x^3 - x^2\right)\frac{d^2y}{dx^2} - \left(x^3 + 2\,x^2 - 2\,x\right)\frac{dy}{dx} + 2\,\left(x^2 + x - 1\right)\,y = 0$$

has the solution $y_1(x) = x\,\mathrm{e}^x$. Use reduction of order to find a solution y_2 such that $\{y_1,\, y_2\}$ is a fundamental solution set for the given ordinary differential equation.

5 Test 1 Make-Up

Problem 41. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x\,y + y}{x}\,\log(x), \quad y(1) = -2.$$

Problem 42. Find the general solution of the ordinary differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \cot(y/x).$$

Problem 43. The ordinary differential equation

$$(9\,y^2 + 18\,x^2y^2 + 4\,xy^3 + x)\,dx + (18\,xy + 6\,y^2)\,dy = 0$$

has an integrating factor depending only on x. Find this integrating factor and then solve the given ordinary differential equation.

Problem 44. Solve the initial value problem

$$\frac{dy}{dx} = \frac{(3\,x + 2\,y)^2 - 3}{2}, \quad y(0) = 1.$$

Problem 45. Given the 1–parameter family of curves

$$x^2 + \alpha\, y^2 = \alpha, \quad (\alpha \text{ is the parameter})$$

find the family of orthogonal trajectories.

Problem 46. A tank initially contains 200 L of brine of concentration $\frac{1}{5}$ g/L of salt. Brine of concentration $\frac{1}{2}$ g/L flows into the tank at 2 L/min. The well–mixed solution is drawn off at the rate 3 L/min. Find the *concentration* of salt in the tank after 100 minutes.

Problem 47. Consider a cup of coffee in a room of temperature A. If T is the temperature of the coffee then

$$\frac{dT}{dt} = -k\,(T - A)$$

where k is a constant depending on the cup. Initially the temperature of the coffee is 183° F, 3 minutes later the temperature is 155° F, and an additional 3 minutes later the temperature is 135° F.

Find the temperature T as a function of time. Compute the temperature of the room.

Problem 48. The ordinary differential equation

$$x\,\frac{d^2y}{dx^2} - x\,\frac{dy}{dx} - y = 0$$

has the solution $y_1(x) = x\,\mathrm{e}^x$. Use reduction of order to find a solution y_2 such that $\{y_1,\, y_2\}$ is a fundamental solution set for the given ordinary differential equation.

6 Test 2

This test originally included a Laplace transform table. The table has been omitted from this archive.

Problem 49. Find the general solution **(in real form)**:

$$\frac{d^2y}{dx^2} - 4\,\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} - 4\,y = 0$$

$$\frac{d^2y}{dx^2} + 4\,y = 0$$

$$\frac{d^2y}{dx^2} - 4\,\frac{dy}{dx} + 4\,y = 0$$

$$\frac{d^2y}{dx^2} - 4\,\frac{dy}{dx} + 13\,y = 0$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6\,y = 0$$

Problem 50. Find the general solution **(in real form)**:

$$x^2\,\frac{d^2y}{dx^2} - 8x\,\frac{dy}{dx} = 0$$

$$x^2\,\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} - 9\,y = 0$$

$$x^2\,\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} + 9\,y = 0$$

$$x^2\,\frac{d^2y}{dx^2} - 5x\,\frac{dy}{dx} + 9\,y = 0$$

$$x^2\,\frac{d^2y}{dx^2} - 5x\,\frac{dy}{dx} + 25\,y = 0$$

$$x^2\,\frac{d^2y}{dx^2} - 6\,y = 0$$

Problem 51. Use variation of parameters to find the general solution of the ordinary differential equation

$$x^2\,\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} - y = 2\,x + 4\,x^{-1}.$$

Problem 52. Use the method of undetermined coefficients (judicious guessing) to find a particular solution:

$$\frac{d^2y}{dx^2} + 3\,\frac{dy}{dx} - 10\,y = x^2\,\mathrm{e}^{2x}$$

$$\frac{d^2y}{dx^2} - 6\,\frac{dy}{dx} + 25\,y = 4x\,\mathrm{e}^{3x}\,\sin(4x)$$

Problem 53. Compute the inverse Laplace transforms:

$$\mathcal{L}^{-1}\left\{\frac{5\,s^2 + 5\,s - 6}{s(s+1)(s-2)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{6\,s^2-7\,s+3}{s^2(s-1)}\right\}$$

Problem 54. Find the Laplace transform of the solution of the following initial value problem:

$$5\,\frac{d^2y}{dt^2}-3\,\frac{dy}{dt}+2\,y=\begin{cases}0 \text{ if } 0\le t<2\\ 2 \text{ if } 2\le t<3\\ 0 \text{ if } 3\le t\end{cases}\qquad y(0)=-2,\quad y'(0)=1.$$

7 Contact Information

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