# Applied Differential Equations – Mth 256

Archive – Spring 1995 Files

*Jan 9, 2001*

This archive contains 6 quizzes and the final exam from Mth 256 Spring 1995. The original test instructions, headers and formatting have not been preserved in order to save space.

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## 1 Test 1

**Problem 1.** A 400.00 L tank initially contains 130.00 L of brine of concentration 0.02000 kg/L. Brine of concentration 0.05000 kg/L flows into the tank at 0.27000 L/sec. The well-mixed solution is drawn off at the rate 0.18000 L/sec. Find the concentration of brine in the tank at the very moment of overflow.

**Problem 2.** A cup of warm coffee is brought into a room. One minute after the coffee is brought into the room the temperature of the coffee is 121° F. Four minutes after the coffee is brought into the room its temperature is observed to be only 83° F. Assume that the temperature of the room is a constant 67° F.

Find the temperature of the coffee when it was initially brought into the room.

Here if $T$ is the temperature of the coffee and $A$ is the temperature of the room then

$$\frac{dT}{dt} = -k\ (T - A)$$

where $k$ is a constant.

**Problem 3.**

**(A)** Solve the initial value problem (IVP):

$$\frac{dy}{dx} = \mathrm{e}^{x+y}, \quad y(0) = 0.$$

**(B)** Find the general solution of the ordinary differential equation:

$$\frac{dy}{dx} = \frac{x\ \cos\left(\frac{y}{x}\right) + y\ \sin\left(\frac{y}{x}\right)}{x\ \sin\left(\frac{y}{x}\right)}.$$

## 2 Test 2

**Problem 4.** Consider the 1–parameter family of exponentials

$$y = \alpha\,\mathrm{e}^{2x}, \quad (\alpha = \text{ parameter}).$$

Find the 1–parameter family of orthogonal trajectories.

**Problem 5.**

**(A):** The differential form

$$\left(8\,x\,y^3 + 3\,y^4 + 3\right)\ dx + \left(6\,x^2\,y^2 + 4\,x\,y^3\right)\ dy$$

has an integrating factor $\mu$ depending only on $x$. Find the integrating factor $\mu$.

**(B):** Solve the ordinary differential equation

$$\left(8\,x\,y^3 + 3\,y^4 + 3\right)\ dx + \left(6\,x^2\,y^2 + 4\,x\,y^3\right)\ dy = 0.$$

## 3 Test 3

**Problem 6.** Find the general solution (in real form):

**(A)** $\dfrac{d^2y}{dx^2} + 2\,\dfrac{dy}{dx} - 8\,y = 0$

**(B)** $\dfrac{d^2y}{dx^2} - 8\,\dfrac{dy}{dx} + 16\,y = 0$

**(C)** $\dfrac{d^2y}{dx^2} - 8\,\dfrac{dy}{dx} = 0$

**(D)** $\dfrac{d^2y}{dx^2} - 8\,y = 0$

**(E)** $\dfrac{d^2y}{dx^2} - 6\,\dfrac{dy}{dx} + 13\,y = 0$

**Problem 7.** Find the general solution (in real form):

**(A)** $x^2\,\dfrac{d^2y}{dx^2} - x\,\dfrac{dy}{dx} - 3\,y = 0$

**(B)** $x^2\,\dfrac{d^2y}{dx^2} + 5x\,\dfrac{dy}{dx} + 4\,y = 0$

**(C)** $x^2\,\dfrac{d^2y}{dx^2} + 3x\,\dfrac{dy}{dx} + 5\,y = 0$

**(D)** $2x^2\,\dfrac{d^2y}{dx^2} + 2x\,\dfrac{dy}{dx} + 3\,y = 0$

**(E)** $2x^2\,\dfrac{d^2y}{dx^2} + 2x\,\dfrac{dy}{dx} - 3\,y = 0$

**Problem 8.** Find the general solution (in real form):

$$\frac{d^2y}{dx^2} + y = \frac{1}{\sin(x)}.$$

## 4 Test 4

### Problem 9.

Use the method of undetermined coefficients to find a particular solution:

**(A)** $\dfrac{d^2y}{dx^2} + 3\,\dfrac{dy}{dx} - y = x^2 + 1$

**(B)** $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x^2 + 1$

### Problem 10.

Use the method of undetermined coefficients to find a particular solution:

**(A)** $\dfrac{d^2y}{dx^2} - y = \mathrm{e}^{x} + \mathrm{e}^{2x}$

**(B)** $\dfrac{d^2y}{dx^2} + 2\,\dfrac{dy}{dx} + 2\,y = x\,\mathrm{e}^{-x}$

### Problem 11.

Use the method of undetermined coefficients to find a particular solution:

**(A)** $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2\,y = \mathrm{e}^{-x}\,\cos(x)$

**(B)** $\dfrac{d^2y}{dx^2} - y = \mathrm{e}^{-x}\,\cos(x)$

**Problem 12.** Consider the ordinary differential equation

$$t^2\,\frac{d^2y}{dt^2} - 6\,y = t^3.$$

Make the substitution

$$t = \mathrm{e}^{x}.$$

Then use the method of undetermined coefficients to solve the resulting ordinary differential equation. Now find a particular solution to the original ordinary differential equation.

## 5 Test 5

### Problem 13.

**(A)** $\mathcal{L}\left\{t^3 e^{5t}\right\} =$

**(B)** $\mathcal{L}\left\{e^{3(t-2)}\right\} =$

**(C)** $\mathcal{L}\left\{(t+2)^3\right\} =$

**(D)** $\mathcal{L}^{-1}\left\{\dfrac{2s^2+8s-2}{(s-1)(s+1)(s+2)}\right\} =$

**(E)** $\mathcal{L}^{-1}\left\{\dfrac{s^2+s-4}{(s+1)(s^2+4)}\right\} =$

### Problem 14.

**(A)** If

$$\mathcal{L}\left\{f(t)\right\} = \frac{s^2+s-2}{s^4-s^3-s+2}$$

compute the LAPLACE transform

$$\mathcal{L}\left\{e^{-2t} f(t)\right\}.$$

**(B)** If

$$f(t) = \begin{cases} 3t, & 0 \le t \le 2 \\ 6, & 2 \le t. \end{cases}$$

compute the LAPLACE transform $\mathcal{L}\left\{f(t)\right\}$.

**Problem 15.** Consider the initial value problem

$$\frac{d^2y}{dt^2} + 3\,\frac{dy}{dt} - 4\,y = t\,e^t, \quad y(0) = -2, \quad y'(0) = 3.$$

Find the Laplace transform of the solution to this initial value problem.

The original test had a Laplace transform table attached here. The table has been appended at the end of this archive.

## 6 Mothers' Weekend Quiz for Mothers

**Problem 16.** Write a brief essay describing your reaction to the O.S.U. campus and what you enjoy most about Mothers' Weekend.

**Problem 17.** A cup of warm coffee is brought into a room. One minute after the coffee is brought into the room the temperature of the coffee is 121° F. Four minutes after the coffee is brought into the room its temperature is observed to be only 83° F. Assume that the temperature of the room is a constant 67° F.

Find the temperature of the coffee when it was initially brought into the room.

Here if $T$ is the (variable) temperature of the coffee and $A$ is the temperature of the room then

$$\frac{dT}{dt} = -k\,(T - A)$$

where $k$ is a constant.

## 7 Final Exam

**Problem 18.** A 100 L tank initially contains 50 L of brine of concentration 3.0 g/L. Brine of concentration 5.0 g/L flows into the tank at 2.0 L/min. The well-mixed solution is drawn off at the rate 1 L/min. Find the concentration of brine in the tank at the very moment of overflow.

**Problem 19.**

**(A):** The differential form

$$\left(8\,x\,y^3 + 3\,y^4 + 3\right)\,dx + \left(6\,x^2\,y^2 + 4\,x\,y^3\right)\,dy$$

has an integrating factor $\mu$ depending only on $x$. Find the integrating factor $\mu$.

**(B):** Solve the ordinary differential equation

$$\left(8\,x\,y^3 + 3\,y^4 + 3\right)\,dx + \left(6\,x^2\,y^2 + 4\,x\,y^3\right)\,dy = 0.$$

**Problem 20.** Find the general solution (in real form):

**(A)** $\dfrac{d^2y}{dx^2} - 6\,\dfrac{dy}{dx} + 9\,y = 0$

**(B)** $\dfrac{d^2y}{dx^2} + 4\,\dfrac{dy}{dx} + 13\,y = 0$

**(C)** $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 6\,y = 0$

**(D)** $x^2 \dfrac{d^2y}{dx^2} - 6\,y = 0$

**(E)** $x^2 \dfrac{d^2y}{dx^2} + 7x\,\dfrac{dy}{dx} + 13\,y = 0$

**Problem 21.** Find the general solution (in real form):

$$\frac{d^2y}{dx^2} + y = \frac{1}{\sin(x)}.$$

**Problem 22.**

Use the method of undetermined coefficients to find a particular solution:

**(A)** $\dfrac{d^2y}{dx^2} - 4\,\dfrac{dy}{dx} + 4y = x\,\mathrm{e}^{2x}$

**(B)** $\dfrac{d^2y}{dx^2} + 2\,\dfrac{dy}{dx} + 2\,y = x\,\mathrm{e}^{x}\,\sin(x)$

**Problem 23.** Consider the ordinary differential equation

$$t^2\,\frac{d^2y}{dt^2} - 6\,y = t^3.$$

Make the substitution

$$t = \mathrm{e}^x.$$

Then use the method of undetermined coefficients to solve the resulting ordinary differential equation. Now find a particular solution to the original ordinary differential equation.

**Problem 24.**

Find the inverse Laplace transforms (in real form):

**(A)** $\mathcal{L}^{-1}\left\{\dfrac{2s+3}{(s-1)(s+1)^2}\right\} =$

**(B)** $\mathcal{L}^{-1}\left\{\dfrac{s^2-2s}{(s+2)(s^2+1)}\right\} =$

**Problem 25.** Consider the initial value problem

$$\frac{d^2y}{dt^2} + 3\,\frac{dy}{dt} - 4\,y = t\,\mathrm{e}^t, \quad y(0) = -2, \quad y'(0) = 3.$$

Find the Laplace transform of the solution to this initial value problem.

The original test had a Laplace transform table attached here. The table has been appended at the end of this archive.

## 8 Contact Information

The contact information below is accurate as of Jan 9, 2001.




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## 9 Laplace Transform Table

The appended Laplace transform table is on the next page.

## Short Table of Laplace Transforms

If $\mathcal{L}\{f(t)\}(s) = F(s)$ then

$$\begin{aligned}
\mathcal{L}\left\{e^{at}f(t)\right\}(s) &= F(s-a) \\
\mathcal{L}\{t^n f(t)\}(s) &= (-1)^n F^{(n)}(s) \\
\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s) &= \int_s^\infty F(r)\,dr \quad \left(\text{if } \frac{f(t)}{t} \text{ integrable at } 0\right) \\
\mathcal{L}\left\{\int_0^t f(r)\,dr\right\}(s) &= \frac{F(s)}{s} \\
\mathcal{L}\left\{\frac{df}{dt}\right\}(s) &= sF(s) - f(0) \quad \left(\text{if } f \text{ cont. on } [0,\infty)\right) \\
\mathcal{L}\left\{\frac{d^2f}{dt^2}\right\}(s) &= s^2F(s) - sf(0) - f'(0) \quad \left(\text{if } f,\ f' \text{ cont. on } [0,\infty)\right) \\
\mathcal{L}\{u(t-a)f(t-a)\}(s) &= e^{-as}F(s) \quad \left(u = \text{ unit step}\right) \\
\mathcal{L}\{f(at)\}(s) &= \frac{1}{a}F(\frac{s}{a}).
\end{aligned}$$

If $\mathcal{L}\{f(t)\}(s) = F(s)$ and $\mathcal{L}\{g(t)\}(s) = G(s)$ then $\mathcal{L}\{(f*g)(t)\}(s) = F(s)G(s)$.

$$\begin{aligned}
\mathcal{L}\left\{e^{at}\right\}(s) &= \frac{1}{s-a} \\
\mathcal{L}\{t^n\}(s) &= \frac{n!}{s^{n+1}} \\
\mathcal{L}\{\cos\omega t\}(s) &= \frac{s}{s^2+\omega^2} \\
\mathcal{L}\{\sin\omega t\}(s) &= \frac{\omega}{s^2+\omega^2} \\
\mathcal{L}\left\{e^{at}\cos\omega t\right\}(s) &= \frac{s-a}{(s-a)^2+\omega^2} \\
\mathcal{L}\left\{e^{at}\sin\omega t\right\}(s) &= \frac{\omega}{(s-a)^2+\omega^2} \\
\mathcal{L}\left\{\sqrt{t}\right\}(s) &= \frac{\sqrt{\pi}}{2s^{3/2}} \\
\mathcal{L}\left\{t^n e^{at}\right\}(s) &= \frac{n!}{(s-a)^{n+1}}.
\end{aligned}$$