Example: Time of Death
Fall 2007 Mth 256
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Assume you are a coroner charged with determining the time of death of a body. Many changes take place in a cadaver and many of them yield estimates of the time of death. We will consider only the cooling of the body as modelled by Newton's Law of Cooling. In particular we will assume that no chemical processes in the body affect the temperature sensibly. In addition we will assume that the ambient temperature A has remained constant since the time of death. Both of these assumptions are generally false and this note should not be taken too seriously.

According to Newton

$$\frac{dT}{dt} = -k(T - A).$$

On solving this equation we find

$$T(t) = C\,\mathrm{e}^{-kt} + A$$

where, as usual, C is a constant to be determined by initial or other conditions. We let t_0 correspond to the time of death. Then substituting t_0 above we obtain

$$C = (B - A)\,\mathrm{e}^{kt_0}$$

where B is the core temperature at the time of death and is normally taken to be 98.6° F. Thus we have

$$T(t) = (B - A)\,\mathrm{e}^{-k(t-t_0)} + A$$

Now we measure the core temperature of the cadaver (usually with a rectal thermometer) at two separate times, $t_1 < t_2$. Let $T_j = T(t_j)$, $j = 1, 2$. Then a little algebra yields

$$\frac{T_1 - A}{T_2 - A} = \mathrm{e}^{k(t_2 - t_1)}$$

which yields

$$k = \frac{1}{t_2 - t_1} \log\left(\frac{T_1 - A}{T_2 - A}\right).$$

In like manner we have

$$k = \frac{1}{t_2 - t_0} \log\left(\frac{B - A}{T_2 - A}\right)$$

and so we obtain

$$t_2 - t_0 = (t_2 - t_1)\,\frac{\log\left(\frac{B-A}{T_2-A}\right)}{\log\left(\frac{T_1-A}{T_2-A}\right)}$$

Note $t_2 - t_0$ is the time since death at the second temperature measurement and $t_2 - t_1$ is just the time between the temperature measurements.

Bogus Example: A body is found in a room with temperature 69° F. A rectal thermometer shows a temperature of 87.6. An hour later the thermometer yields 86.3. Find the time of death. (Ans: 7.4 hours prior to the second measurement). Suppose the assumption that $B = 98.6°$ F is incorrect and that the living person had a normal temperature of only 97.5. What is your new estimate? (Ans: 6.9 hours). Alternately suppose the person had a fever of 100.5. What is your new estimate? (Ans: 8.3 hours)