Mth 256 Test 1 | Fall 2007 | Name:

Bent Petersen `256f2007-test1.tex`
Date: Oct 31 2007, 1500. Location: KIDD 364. Time: 50 m in.

- A scantron is provided with this test. Fill in your ID information on the scantron now. Also enter your name on this test in the space provided above. Do not fold, staple or tear, etc., the scantron. Return both the entire test and the scantron (separately).
- This test consists of multiple-choice problems and one work-out problem. Fill in the answers to the multiple-choice problems in the boxes provided and on the scantron. You are expected to fill in the scantron properly. Depending on your solution methods your answers may appear in a different form from the ones provided on the test. You are expected to be able to provide the appropriate manipulations to identify the correct answer.
- For the work-out problem you are expected to show some well-chosen work (on the test). Otherwise you will not receive full credit.
- You may use one 8.5×11 inch note sheet prepared in advance. You may write on both sides of your note sheet. Note sheets may not be shared. If you do not bring a note sheet you will have to do without any help notes. You may not use any books, notebooks, additional note sheets nor note cards.
- You may use a simple scientific calculator or a modest graphics calculator on this test and you are expected to have one available. An overly elaborate calculator, laptop, handheld or notebook computer, or any device capable of extensive symbolic manipulation (other than your own brain) will not be allowed. Calculators and other equipment may not be shared.
- During the test be sure to check the board occasionally for corrections. Note $\log(x)$ means the natural logarithm of x.
- There are 9 multiple-choice problems worth 10 points each, and 1 work-out problem worth 20 points. The total number of points is 110 points. The number of problems is 10.

Multiple-choice problems: 9 problems, 10 points each.

Problem 1. Solve the initial value problem

$$\frac{dy}{dt} - y = 1$$

with initial condition $y(0) = -2$. Then compute $y(\log(2))$.

A.) 0 **B.)** 1
C.) 3 **D.)** -3 **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 1).*

Problem 2. Find the general solution $y(t)$ of the ordinary differential equation

$$\frac{dy}{dt} + \tan(t)\, y = \cos(t).$$

on the interval $(-\pi/2, \pi/2)$, and then calculate the limit of $y(t)$ as t goes to $\pi/2$.

A.) -1 **B.)** 0
C.) 1 **D.)** 2 **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 2).*

Problem 3. Solve the initial value problem

$$\frac{dy}{dt} = y^2 + ty^2, \quad y(0) = -2.$$

Then compute $y(2)$.

A.) $-2/9$ **B.)** $-1/2$
C.) $-1/8$ **D.)** $3/4$ **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 3).*

Problem 4. Solve the initial value problem

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6\, y = 0, \quad y(0) = -1, y'(0) = 5.$$

Then compute $y(\log(2))$.

A.) $57/40$ **B.)** $17/9$
C.) $51/5$ **D.)** $479/135$ **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 4).*

Problem 5. Solve the initial value problem

$$\frac{d^2y}{dt^2} + 6\,\frac{dy}{dt} + 9\, y = 0, \quad y(0) = 0, y'(0) = 1.$$

A.) $y(t) = \mathrm{e}^{-3t}$ **B.)** $y(t) = t\mathrm{e}^{-3t}$
C.) $y(t) = 3t\mathrm{e}^{-3t}$ **D.)** $y(t) = \mathrm{e}^{-3t} - 6t\mathrm{e}^{-3t}$ **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 5).*

Problem 6. Solve the initial value problem

$$\frac{d^2y}{dt^2} + 2\,\frac{dy}{dt} + 10\,y = 0, \quad y(0) = 1, y'(0) = -1.$$

A.) $\cos(3t) - (1/3)\sin(3t)$ **B.)** $\cos(3t)$

C.) $\mathrm{e}^{-t}\cos(3t) - \mathrm{e}^{-t}\sin(3t)$ **D.)** $\mathrm{e}^{-t}\cos(3t)$ **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 6).*

Problem 7. Solve the initial value problem

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0, \quad y(0) = 1, y'(0) = -2, y''(0) = 4.$$

A.) e^{-t} **B.)** e^{t}

C.) e^{-2t} **D.)** e^{2t} **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 7).*

Problem 8. The ordinary differential equation

$$y + \left(y^2 - x\right)\frac{dy}{dx} = 0$$

Has an integrating factor depending only on y. Find one.

A.) y^2 **B.)** y

C.) y^{-1} **D.)** y^{-2} **E.)** None of the foregoing.

☐ ←*Mark answer here and on the scantron* *(Problem 8).*

Problem 9. A cup of coffee initially at temperature $T_0 = 182°$F is brought into a room of temperature $A = 69°$F. Then according to Newton's law of cooling

$$\frac{dT}{dt} = -k(T - A)$$

where k is a constant depending on the thermal properties of the cup. The heat capacity of the room (compared to the coffee) is so large that we may regard A as constant. After 7 minutes the temperature of the coffee is $143°$. What will be the temperture of the coffee an additional 8 minutes later? Chose the closest number from the list below.

A.) $158°$ **B.)** $139°$

C.) $127°$ **D.)** $115°$ **E.)** $103°$

☐ ←*Mark answer here and on the scantron* *(Problem 9).*

Work-out problems: 1 problem(s), 20 points each.

Problem 10. A 100 gallon tank initially contains 50 gallons of brine of concentration 3 oz/gal salt. Brine of concentration 2 oz/gal salt flows into the tank at 4 gallons per minute. The well-mixed solution is pumped out at 3 gallons per minute. Find the concentration of salt in the brine in the tank at the very moment of overflow.

Show clear and careful work for partial credit.

Use the backs of the test pages for scratch work.