

This file contains a few sample problems. These problems were thrown together rather quickly. Expect some errors! This is not a complete set of problems. I will try to find time to produce more. Meanwhile take a look at the old tests archive.

Problem 1. What is the radius of the smallest ball centered at $(6, -8, 2\sqrt{11})$ which contains the origin?

Problem 2. Write an equation for the set of points that have distance 4 from the line $x = -1 + 2t$, $y = 2 + 3t$, and $z = 2 - t$.

Problem 3. Given

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c},$$

conclude

$$\vec{a} \times (\vec{b} \times \vec{c}) - (\vec{a} \times \vec{b}) \times \vec{c} = (\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{b})\vec{c}.$$

Can you show the “given” part?

Problem 4. Find the cosine of the angle θ between the vectors $\vec{a} = \langle 2, -1, 2 \rangle$ and $\vec{b} = \langle 3, 12, -4 \rangle$.

Problem 5. Find an equation of the plane through the points $(3, 2, 1)$, $(0, 2, -4)$ and $(1, 4, 2)$.

Problem 6. Find the perpendicular distance from the point $(3, 1, 1)$ to the plane $3x + 3y - 2z = 5$.

Problem 7. Find the parametric equation of the line through the point $(1, 3, 2)$ and parallel to the vector $\vec{v} = \langle -2, 1, 0 \rangle$.

Problem 8. Find the parametric equation of the line through the points $P_1 = (-1, 3, 7)$ and $P_2 = (4, 8, -3)$. Then find the point $2/5$ of the way along the line from P_1 to P_2 .

Problem 9. Find an equation of the plane through the point $(2, -2, 5)$ and parallel to the vectors $\vec{a} = \langle 2, 1, 0 \rangle$ and $\vec{b} = \langle 1, 3, -1 \rangle$.

Problem 10. Let $a > 0$. Show that the parametric curve

$$x = a \frac{t^2 - 1}{t^2 + 1}, \quad y = a \frac{2t}{t^2 + 1}$$

is a circle with center at the origin. What is the radius? Find the velocity and the unit tangent. Find the acceleration.

Problem 11. Let $a > 0$. Show that the parametric curve

$$x = a \frac{3t}{t^3 + 1}, \quad y = a \frac{3t^2}{t^3 + 1}$$

is a parametrization of the folium of Descartes

$$x^3 + y^3 = 3axy.$$

Find the velocity and the unit tangent. Find the acceleration.

Problem 12. Find an equation of the plane of intersection of the spheres $x^2 + y^2 + z^2 = 1$ and $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 = 1$. Find the distance of the plane to the origin.