Vector Calculus I – Mth 254

Archive – Winter 2000 Files

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This archive contains the midterm and the final exam from Mth 254 Winter 2000. The original test instructions, headers and formatting have not been preserved. These multiple choice tests made use of a scantron.

Contents

1 Mid-term

Problem 1. Find the angle between the vectors $\vec{a} = \langle -1, 2, -2 \rangle$ and $\vec{b} = \langle 0, 24, 7 \rangle$

A.) $\frac{34}{75}$

B.) $\arccos(\frac{34}{75})$

C.) $\arcsin(\frac{34}{75})$

D.) $\arccos(34)$

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 1).*

Problem 2. Find a vector orthogonal (perpendicular, normal) to the plane which contains the points $(0, 1, 2)$, $(2, 1, 0)$ and $(1, 0, -1)$.

A.) $\langle 1, 2, 1 \rangle$

B.) $\langle 2, 0, -2 \rangle$

C.) $\langle 1, 1, -3 \rangle$

D.) $\langle -2, 4, -2 \rangle$

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 2).*

Problem 3. Find the (unsigned) distance from the plane $3x + 12y + 4z = 9$ to the point $(4, 2, 3)$.

A.) 48/13
B.) 57/13
C.) 3
D.) 39
E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 3).*

Problem 4. If

$$f(x, y) = \frac{3x^2y + 2y^2x}{x^2 + y^2}$$

find $\dfrac{\partial f}{\partial y}(1, 1)$.

A.) 1
B.) 2
C.) 3
D.) 6
E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 4).*

Problem 5. Find the equation of the tangent plane to the graph of $f(x, y) = x^4y^2 - x^2y^4$ at $(2, 1)$.

A.) $28x + 16y - z = 60$
B.) $28x + 16y + z = 60$
C.) $28x + 16y - z = 72$
D.) $28x + 16y + z = 72$
E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 5).*

Problem 6. If $f(x, y) = \sqrt{x^3 + y^3}$ then $f(2, 1) = 3$. Use differentials (relative to the point $(2, 1)$) to estimate $\sqrt{(2.0100)^3 + (0.9700)^3}$. You must provide the answer obtained by using differentials to estimate.

A.) 3.0060

B.) 3.0055

C.) 3.0050

D.) 3.0044

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 6).*

Problem 7. Let $c > 0$ be a constant, let f be a smooth function of one variable, let $s = x - ct$ and let $w = f(s)$. Then

$$\frac{\partial^2 w}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} =$$

A.) $2w$

B.) $2f''(s)$

C.) $2(f'(s))^2$

D.) 0

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 7).*

Problem 8. If $f(x, y) = \cos\left(\frac{x}{x+y}\right)$ then

$$\frac{\partial f}{\partial y} - \frac{\partial f}{\partial x} =$$

A.) $\sin(x/(x+y)) \times (x-y)/(x+y)^2$

B.) $\sin(x/(x+y)) \times 1/(x+y)^2$

C.) $\sin(x/(x+y)) \times 1/(x+y)$

D.) 0

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 8).*

Problem 9. Let R be the rectangle $\{\,(x, y) \mid 1 \le x \le 2, 2 \le y \le 4\,\}$. Compute

$$\iint_R x^2 y \, \mathrm{d}x \, \mathrm{d}y.$$

A.) 14

B.) 28

C.) 42

D.) 84

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 9).*

Problem 10. Let R be the rectangle $\{ (x, y) \mid 1 \le x \le 2, 1 \le y \le 3 \}$. Compute

$$\iint\limits_R x\mathrm{e}^{xy}\,\mathrm{d}x\,\mathrm{d}y.$$

A.) $\frac{65}{27}\mathrm{e}^9 - \frac{140}{27}\mathrm{e}^3 - \mathrm{e}$

B.) $\frac{1}{3}\mathrm{e}^6 - \frac{1}{3}\mathrm{e}^3 + 2\mathrm{e}^2 - 2\mathrm{e}$

C.) $\frac{65}{27}\mathrm{e}^9 - \frac{140}{27}\mathrm{e}^3 + \mathrm{e}$

D.) $\frac{1}{3}\mathrm{e}^6 - \frac{1}{3}\mathrm{e}^3 - \mathrm{e}^2 + \mathrm{e}$

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 10).*

2 Final Exam

Problem 11. Let $Ax = b$ be a system of 3 linear equations in 3 unknowns. After row reducing the augmented matrix $[A\ b]$ we obtain the completely reduced row echelon Gauss–Jordan canonical form matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

A.) The system $Ax = b$ has no solutions (i.e., is inconsistent)

B.) The system $Ax = b$ has a unique solution

C.) The system $Ax = b$ has a one-parameter family of solutions

D.) The system $Ax = b$ has a two, or more, parameter family of solutions

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 11).*

Problem 12. Let $Ax = b$ be a system of 3 linear equations in 3 unknowns. After row reducing the augmented matrix $[A\ b]$ we obtain the completely reduced row echelon Gauss–Jordan canonical form matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

A.) The system $Ax = b$ has no solutions (i.e., is inconsistent)

B.) The system $Ax = b$ has a unique solution

C.) The system $Ax = b$ has a one-parameter family of solutions

D.) The system $Ax = b$ has a two, or more, parameter family of solutions

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 12).*

Problem 13. Let $Ax = b$ be a system of 3 linear equations in 3 unknowns. After row reducing the augmented matrix $[A\ b]$ we obtain the completely reduced row echelon Gauss–Jordan canonical form matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

A.) The system $Ax = b$ has no solutions (i.e., is inconsistent)

B.) The system $Ax = b$ has a unique solution

C.) The system $Ax = b$ has a one-parameter family of solutions

D.) The system $Ax = b$ has a two, or more, parameter family of solutions

E.) None of the above.

☐ ← *Write letter corresponding to your answer here and mark it on the scantron (Problem 13).*

Problem 14. Let $Ax = b$ be a system of 3 linear equations in 3 unknowns. After row reducing the augmented matrix $[A\ b]$ we obtain the completely reduced row echelon Gauss–Jordan canonical form matrix

$$A = \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

A.) The system $Ax = b$ has no solutions (i.e., is inconsistent)

B.) The system $Ax = b$ has a unique solution

C.) The system $Ax = b$ has a one-parameter family of solutions

D.) The system $Ax = b$ has a two, or more, parameter family of solutions

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 14).*

Problem 15. If

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

compute $I + J^2$.

A.) J

B.) $2I$

C.) $I - J$

D.) 0

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 15).*

Problem 16. Given that $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 9 & 0 & 1 \\ 0 & 7 & 0 \\ 0 & 2 & 8 \end{bmatrix}$ find the corresponding eigenvalue.

A.) 5

B.) 6

C.) 7

D.) 8

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 16).*

Problem 17. Given 8 is an eigenvalue of the matrix $\begin{bmatrix} 9 & 0 & 1 \\ 0 & 7 & 0 \\ 0 & 2 & 8 \end{bmatrix}$ find a corresponding eigenvector.

A.) $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

B.) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

C.) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

D.) 0

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 17).*

Problem 18. Find the surface area of the part of the paraboloid $z = x^2 + y^2$ which lies above the disk $x^2 + y^2 \leq 1$.

A.) $\frac{\pi}{2}\left(5\sqrt{5} - 1\right)$
B.) $\frac{\pi}{4}\left(5\sqrt{5} - 1\right)$
C.) $\frac{\pi}{6}\left(5\sqrt{5} - 1\right)$
D.) $\frac{\pi}{6}\left(\sqrt{5} - 1\right)$
E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 18).*

Problem 19. Find the volume under the paraboloid $z = x^2 + y^2$ and above the region $x^2 + y^2 \leq 2x$.

A.) $3\pi/2$
B.) $16\pi/5$
C.) 3π
D.) 2π
E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 19).*

Problem 20. Evaluate

$$\iiint\limits_{\Omega} x \;\, \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

where $\Omega = \{\,(x, y, z) \;\mid\; 0 \leq z \leq 1 - xy, 0 \leq x \leq 1, 0 \leq y \leq 1\,\}$.

A.) $1/2$
B.) $1/3$
C.) $3/4$
D.) $2/3$
E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 20).*

Problem 21. Let $B = \left\{\,(x, y, z) \;\mid\; x^2 + y^2 + z^2 \leq 2\,\right\}$. Evaluate

$$\iiint\limits_{B} 4\left(x^2 + y^2 + z^2\right)^{5/2} \;\, \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

A.) $\frac{32}{3}\pi$

B.) 32π

C.) 256π

D.) $\frac{64}{7}\sqrt{2}\pi$

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 21).*

Problem 22. Evaluate the triple integral

$$\iiint\limits_T \left(x^2+y^2+z\right)^{3/2} \; \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

where $T = \left\{ (x,y,z) \;\mid\; x^2+y^2 \le 1,\, 0 \le z \le 2 \right\}$.

A.) $\frac{\pi}{15}\left(30\log(1+\sqrt{3}) + 48\sqrt{3} - 15\log(2) - 2\right)$

B.) $\frac{63\pi}{16}$

C.) $\frac{4\pi}{35}\left(27\sqrt{3} - 8\sqrt{2} - 1\right)$

D.) $\frac{74\pi}{11}$

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 22).*

Problem 23. Suppose $w(t) = f(x,y)$ where $x = g(t)$ and $y = h(t)$. Use the information in the table below to compute $w'(0)$.

$g(0)=1$	$g(1)=0$	$g(2)=-1$	$h(0)=2$	$h(1)=3$	$h(2)=4$
$g'(0)=2$	$g'(1)=-2$	$g'(2)=1$	$h'(0)=1$	$h'(1)=-2$	$h'(2)=2$
$f_x(0,1)=3$	$f_x(1,2)=-2$	$f_y(0,1)=2$	$f_y(1,2)=3$	$f(0,1)=1$	$f(1,2)=3$

A.) -7

B.) -6

C.) 7

D.) 6

E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 23).*

Problem 24. Find the slope of the tangent line to the curve

$$x^3+y^3-3xy-x+2y=6$$

at the point $(1, 2)$.

A.) $\frac{3}{11}$
B.) $\frac{4}{11}$
C.) $\frac{4}{9}$
D.) $\frac{5}{9}$
E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 24).*

Problem 25. The matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$

has determinant 160. A is

A.) non-invertible
B.) diagonizable
C.) non-diagonizable
D.) upper-triangular
E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 25).*

Problem 26. The tangent plane to the surface

$$xyz = 16$$

at $(2, 2, 4)$ is given by

A.) $x + y - z = 4$
B.) $x + y + z = 8$
C.) $2x + 2y - z = 4$
D.) $2x + 2y + z = 12$
E.) None of the above.

☐ ←*Write letter corresponding to your answer here and mark it on the scantron (Problem 26).*

3 Contact Information

The contact information below is accurate as of Jan 9, 2001.

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