254u2000_sg_eigenvects.mws
Mth 254 Summer 2000

Answers to section 22 of the Spring Term edition of the Mth 254 Study Guide.

Note the eigenvects() command returns a list of objects of the form

[eigenvalue, multiplicity, (eigenvectors)]

I included the eigenvals() command, which returns just the eigenvalues, because its output is easier to read in those cases where one just needs to know the eigenvalues.

```
> restart;
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
> A01:=matrix(2,2,[6,2,1,5]);
```

$$A01 := \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}$$

```
> eigenvals(A01);
```

$$4, 7$$

```
> eigenvects(A01);
```

$$[4, 1, \{[-1, 1]\}], [7, 1, \{[2, 1]\}]$$

```
> A02:=matrix(2,2,[-4,1,2,-3]);
```

$$A02 := \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix}$$

```
> eigenvals(A02);
```

$$-5, -2$$

```
> eigenvects(A02);
```

$$[-2, 1, \{[1, 2]\}], [-5, 1, \{[-1, 1]\}]$$

```
> A03:=matrix(2,2,[4,-1,5,2]);
```

$$A03 := \begin{bmatrix} 4 & -1 \\ 5 & 2 \end{bmatrix}$$

```
> eigenvals(A03);
```

$$3 + 2\,I, 3 - 2\,I$$

```
> eigenvects(A03);
```

$$[3 + 2\,I, 1, \{[1, 1 - 2\,I]\}], [3 - 2\,I, 1, \{[1, 1 + 2\,I]\}]$$

```
> A04:=matrix(2,2,[-5,1,-5,-3]);
```

$$A04 := \begin{bmatrix} -5 & 1 \\ -5 & -3 \end{bmatrix}$$

```
> eigenvals(A04);
```

$$-4 + 2\,I, -4 - 2\,I$$

```
> eigenvects(A04);
```

$$\left[-4+2\,I,\,1,\,\left\{\left[\frac{1}{5}-\frac{2}{5}\,I,\,1\right]\right\}\right],\left[-4-2\,I,\,1,\,\left\{\left[\frac{1}{5}+\frac{2}{5}\,I,\,1\right]\right\}\right]$$

```
> A05:=matrix(2,2,[-6,2,2,-3]);
```

$$A05 := \begin{bmatrix} -6 & 2 \\ 2 & -3 \end{bmatrix}$$

```
> eigenvals(A05);
```

$$-7, -2$$

```
> eigenvects(A05);
```

$$[-2, 1, \{[1, 2]\}], [-7, 1, \{[-2, 1]\}]$$

```
> A06:=matrix(2,2,[6,2,2,3]);
```

$$A06 := \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

```
> eigenvals(A06);
```

$$2, 7$$

```
> eigenvects(A06);
```

$$[2, 1, \{[1, -2]\}], [7, 1, \{[2, 1]\}]$$

```
> A07:=matrix(3,3,[5,2,2,2,-2,0,2,0,-2]);
```

$$A07 := \begin{bmatrix} 5 & 2 & 2 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}$$

```
> eigenvals(A07);
```

$$6, -3, -2$$

```
> eigenvects(A07);
```

$$[-3, 1, \{[1, -2, -2]\}], [-2, 1, \{[0, -1, 1]\}], [6, 1, \{[4, 1, 1]\}]$$

```
> A08:=matrix(3,3,[5,3,3,3,2,0,3,0,2]);
```

$$A08 := \begin{bmatrix} 5 & 3 & 3 \\ 3 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

```
> eigenvals(A08);
```

$$2, 8, -1$$

```
> eigenvects(A08);
```

$$[2, 1, \{[0, -1, 1]\}], [8, 1, \{[2, 1, 1]\}], [-1, 1, \{[-1, 1, 1]\}]$$

```
> A09:=matrix(3,3,[-7,0,3,0,-7,2,3,2,5]);
```

$$A09 := \begin{bmatrix} -7 & 0 & 3 \\ 0 & -7 & 2 \\ 3 & 2 & 5 \end{bmatrix}$$

```
> eigenvals(A09);
```

$$6, -8, -7$$

```
> eigenvects(A09);
```

$$[-8, 1, \{[-3, -2, 1]\}], \left[-7, 1, \left\{\left[1, \frac{-3}{2}, 0\right]\right\}\right], \left[6, 1, \left\{\left[\frac{3}{2}, 1, \frac{13}{2}\right]\right\}\right]$$

```
> A10:=matrix(3,3,[1,0,1,0,1,2,1,2,5]);
```

$$A10 := \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

```
> eigenvals(A10);
```

$$0, 1, 6$$

```
> eigenvects(A10);
```

$$[6, 1, \{[1, 2, 5]\}], [1, 1, \{[-2, 1, 0]\}], [0, 1, \{[-1, -2, 1]\}]$$

```
>
```

Problem 11

```
> form11:=-6*x^2+4*x*y-3*y^2;
```

$$form11 := -6\,x^2 + 4\,x\,y - 3\,y^2$$

```
> A11:=hessian(form11/2,[x,y]);
```

$$A11 := \begin{bmatrix} -6 & 2 \\ 2 & -3 \end{bmatrix}$$

```
> eigenvals(A11);
```

$$-7, -2$$

```
> V11:=eigenvects(A11);
```

$$V11 := [-7, 1, \{[-2, 1]\}], [-2, 1, \{[1, 2]\}]$$

```
> zz:=op(1,V11[1][3]): col1:=%/norm(%,2);
```

$$col1 := \frac{1}{5}[-2, 1]\sqrt{5}$$

```
> zz:=op(1,V11[2][3]): col2:=%/norm(%,2);
```

$$col2 := \frac{1}{5}[1, 2]\sqrt{5}$$

```
> S11:=augment(col1,col2);
```

$$S11 := \begin{bmatrix} -\frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \\ \frac{1}{5}\sqrt{5} & \frac{2}{5}\sqrt{5} \end{bmatrix}$$

```
> J11:=evalm(inverse(S11) &* A11 &* S11);
```

$$J11 := \begin{bmatrix} -7 & 0 \\ 0 & -2 \end{bmatrix}$$

```
> canonf11:=evalm(matrix(1,2,[u,v]) &* J11 &*
  matrix(2,1,[u,v]))[1,1];
```

$$canonf11 := -7\,u^2 - 2\,v^2$$

Problem 12

```
> form12:=6*x^2+4*x*y+3*y^2;
```

$$form12 := 6\,x^2 + 4\,x\,y + 3\,y^2$$

```
> A12:=hessian(form12/2,[x,y]);
```

$$A12 := \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

```
> eigenvals(A12);
```

$$2,\ 7$$

```
> V12:=eigenvects(A12);
```

$$V12 := [7,\ 1,\ \{[2,\ 1]\}],\ [2,\ 1,\ \{[1,\ -2]\}]$$

```
> zz:=op(1,V12[1][3]): col1:=%/norm(%,2);
```

$$col1 := \frac{1}{5}\,[2,\ 1]\,\sqrt{5}$$

```
> zz:=op(1,V12[2][3]): col2:=%/norm(%,2);
```

$$col2 := \frac{1}{5}\,[1,\ -2]\,\sqrt{5}$$

```
> S12:=augment(col1,col2);
```

$$S12 := \begin{bmatrix} \frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \\ \frac{1}{5}\sqrt{5} & -\frac{2}{5}\sqrt{5} \end{bmatrix}$$

```
> J12:=evalm(inverse(S12) &* A12 &* S12);
```

$$J12 := \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix}$$

```
> canonf12:=evalm(matrix(1,2,[u,v]) &* J12 &*
  matrix(2,1,[u,v]))[1,1];
```

$$canonf12 := 7\,u^2 + 2\,v^2$$

```
>
```