Vector Calculus I – Mth 254

Archive – Spring 1996 Files

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This document contains two quizzes and the final exam from Mth 254 Spring 1996. The original test formatting has not been preserved.

Contents

1 Quiz 1

Problem 1. Let P_0 be the point with position vector $\vec{r}_0 =< 3, 2, 2 >$. Let P_1 be the point with position vector $\vec{r}_1 =< 1, 3, 2 >$.

(A) Find the parametric vector equation (or parametric scalar equations) of the line L through the point P_1 and parallel to the vector $\vec{w} =< 1, -3, 2 >$.

(B) Find the equation of the plane Π through P_0 and perpendicular to L.

(C) Find the point P_2 of intersection of L and Π.

(D) Find the distance from P_0 to L.

Problem 2. If $z = f(x,y)$, $x = \phi(u,v)$ and $y = \psi(u,v)$ where f, ϕ and ψ are differentiable functions and we are given the following data:

$f(1,2)=4$	$\phi(0,1)=1$	$\phi(1,2)=-1$	$\psi(0,1)=2$	$\psi(1,2)=4$
$f(0,1)=-2$	$\frac{\partial f}{\partial x}(1,2)=3$	$\frac{\partial f}{\partial x}(0,1)=-4$	$\frac{\partial f}{\partial y}(1,2)=-2$	$\frac{\partial f}{\partial y}(0,1)=5$
$\frac{\partial \phi}{\partial u}(1,2)=1$	$\frac{\partial \phi}{\partial u}(0,1)=-1$	$\frac{\partial \psi}{\partial u}(1,2)=-3$	$\frac{\partial \psi}{\partial u}(0,1)=2$	$\frac{\partial \phi}{\partial v}(1,2)=7$
$\frac{\partial \phi}{\partial v}(0,1)=6$	$\frac{\partial \psi}{\partial v}(1,2)=4$	$\frac{\partial \psi}{\partial v}(0,1)=3$	$\frac{\partial \phi}{\partial v}(0,4)=8$	$\frac{d\psi}{dv}(0,4)=-5$

Compute

$$\frac{\partial z}{\partial u}(0,1).$$

Problem 3. Compute the following partial derivatives:

$$\textbf{(A)}\ \frac{\partial}{\partial x}\left(\arctan(\frac{y}{x})\right) \qquad \textbf{(B)}\ \frac{\partial}{\partial u}\log(uv) \qquad \textbf{(C)}\ \frac{\partial}{\partial y}\left(\frac{x+y^2}{x^2+y^2}\right).$$

Problem 4. Suppose $z = f(x,y)$ is differentiable at $(3,2)$, $f(3,2) = 4$, $\frac{\partial z}{\partial x}(3,2) = -3$ and $\frac{\partial z}{\partial y}(3,2) = 2$.

(A) Find the equation of the tangent plane to the graph of f at the point $(3,2,4)$.

(B) Use differentials to approximate $f(3.1, 1.9)$.

Problem 5. **(A)** Change the order of integration in the iterated integral

$$\int_0^2 \int_0^{x^2} f(x,y)\,dydx.$$

(B) Evaluate the iterated integral $\displaystyle\int_{-1}^2 \int_0^2 \left(x^2 - y^2\right)\,dxdy.$

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2 Quiz 2

Problem 6. Use polar coordinates to evaluate the double integral

$$\iint\limits_{\Omega} \frac{y}{\sqrt{x^2+y^2}}\,dxdy$$

where Ω is the semi–cardioid described in polar coordinates by $0 \leq r \leq 1+\cos\theta$, $0 \leq \theta \leq \pi$.

Problem 7. Find the *surface area* of that part of the cone $z = \sqrt{x^2+y^2}$ which lies above the region Ω bounded by the curve $x^2+y^2 = 2x$.

Problem 8. Let Ω be the region bounded by the two spheres $x^2+y^2+z^2 = 12$ and $x^2+y^2+z^2 = 4\sqrt{3}\,z$. Evaluate the triple integral

$$\iiint\limits_{\Omega} z\,dxdydz.$$

Problem 9. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a 2×2 matrix such that

$$A\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 2 & -2 \end{bmatrix}.$$

Compute A.

Problem 10. Compute the determinant of the 3×3 matrix

$$A = \begin{bmatrix} 3 & x & -2 \\ y & 2 & -1 \\ 4 & 1 & 1 \end{bmatrix}$$

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3 Final Exam

Problem 11. The matrix

$$A = \begin{pmatrix} -2 & 1 & 1 \\ -6 & 1 & 3 \\ -12 & -2 & 8 \end{pmatrix}$$

has eigenvalues 1, 2, and 4. Find the corresponding eigenvectors of A. Make sure you indicate which eigenvector goes with which eigenvalue.

Problem 12. The matrix

$$A = \begin{pmatrix} -14 & -30 & 42 \\ 24 & 49 & -66 \\ 12 & 24 & -32 \end{pmatrix}$$

has eigenvectors

$$\vec{u}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}, \quad \text{and} \quad \vec{u}_3 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}.$$

Find the eigenvalues corresponding to each of these eigenvectors. (Hint: Think! This problem can be done with very little computation). Make sure you indicate which eigenvector goes with which eigenvalue.

Problem 13. Suppose $z = f(x,y)$ is differentiable at $(1,2)$, $f(1,2) = -2$, $\dfrac{\partial z}{\partial x}(1,2) = -4$ and $\dfrac{\partial z}{\partial y}(1,2) = -3$.

(A) Find the equation of the tangent plane to the graph of f at the point $(1,2,-2)$.

(B) Use differentials to approximate $f(0.9, 2.1)$.

Problem 14. Change the order of integration in the iterated integral

$$\int_{-2}^{2} \int_{4}^{8-x^2} f(x,y)\, dydx.$$

Problem 15. Let a, b, and c be real numbers, not all zero, and let d be a real number. Then

$$ax + by + cz = d$$

is the equation of a plane Π in Euclidean 3–space.

(A) Find the parametric equations of the line through the origin and perpendicular to the plane Π.

(B) Find the perpendicular distance from the plane Π to the origin.

Problem 16. Find the *surface area* of that part of the sphere $x^2 + y^2 + z^2 = 4$ which lies above the region Ω bounded by the curve $x^2 + y^2 = 2x$.

Problem 17. Let Ω be the region inside the sphere $x^2 + y^2 + z^2 = 169$ and above the plane $z = 5$. Evaluate the triple integral

$$\iiint\limits_{\Omega} z\, dxdydz.$$

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