Linear Algebra Supplement
Selected Solutions

Dec 6 1996

Mth 254 Fall 1996

Eigenvalues and eigenvectors for some of the matrices in Anselone and Lee, Mth 254 Linear Algebra Supplement, 1966.

The solutions below were provided by Maple V4 (sometimes with a bit of coaxing). These solutions are provided to allow you to check your own work. You do not need to learn Maple (though you may want to learn it after looking below).

```
> restart;
> with(linalg):
```

```
Warning, new definition for norm

Warning, new definition for trace
```

Assignment on page 46

Number 1

```
> A:=matrix(2,2,[[5,-1],[3,1]]);
```

$$A := \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$$

```
> eigenvals(A);
```

$$2, 4$$

```
> E:=eigenvects(A);
```

$$E := [2, 1, \{[1, 3]\}], [4, 1, \{[1, 1]\}]$$

```
> S:=transpose(matrix(2,2,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

==

Number 2

```
> A:=matrix(2,2,[[3,-2],[4,-1]]);
```

$$A := \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$

```
> eigenvals(A);
```

$$1 + 2\,I,\ 1 - 2\,I$$

```
> E:=eigenvects(A);
```

$$E := [1 + 2\,I,\ 1,\ \{[1,\ 1 - I]\}],\ [1 - 2\,I,\ 1,\ \{[1,\ 1 + I]\}]$$

```
> S:=transpose(matrix(2,2,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 1 & 1 \\ 1 - I & 1 + I \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{1}{2} - \frac{1}{2}\,I & \frac{1}{2}\,I \\ \frac{1}{2} + \frac{1}{2}\,I & -\frac{1}{2}\,I \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} 1 + 2\,I & 0 \\ 0 & 1 - 2\,I \end{bmatrix}$$

==

Number 3

```
> A:=matrix(3,3,[[1,-1,4],[3,2,-1],[2,1,-1]]);
```

$$A := \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$$

```
> eigenvals(A);
```

$$1,\ 3,\ -2$$

```
> E:=eigenvects(A);
```

$$E := [3,\ 1,\ \{[1,\ 2,\ 1]\}],\ [-2,\ 1,\ \{[-1,\ 1,\ 1]\}],\ [1,\ 1,\ \{[1,\ -4,\ -1]\}]$$

```
> S:=transpose(matrix(3,3,[op[1](E[1][3]),op[1] (E[2][3]),op[1](E[3][3])]));
```

$$S := \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -4 \\ 1 & 1 & -1 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{2} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

====================================

Number 4

```
> A:=matrix(2,2,[[1,-4],[4,-7]]);
```

$$A := \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix}$$

```
> eigenvals(A);
```

$$-3,\ -3$$

```
> E:=eigenvects(A);
```

$$E := [-3,\ 2,\ \{[1,\ 1]\}]$$

A is not diagonizable.

====================================

Number 5

```
> A:=matrix(3,3,[[1,0,0],[2,1,-2],[3,2,1]]);
```

$$A := \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix}$$

```
> eigenvals(A);
```

$$1,\ 1+2\,I,\ 1-2\,I$$

```
> E:=eigenvects(A);
```

$$E := [1+2\,I,\ 1,\ \{[0,\ 1,\ -I]\}],\ [1-2\,I,\ 1,\ \{[0,\ 1,\ I]\}],\ [1,\ 1,\ \{\left[1,\ \frac{-3}{2},\ 1\right]\}]$$

```
> S:=transpose(matrix(3,3,[op[1](E[1][3]),op[1] (E[2][3]),op[1](E[3][3])]));
```

$$S := \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & \frac{-3}{2} \\ -I & I & 1 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{3}{4} - \frac{1}{2}I & \frac{1}{2} & \frac{1}{2}I \\ \frac{3}{4} + \frac{1}{2}I & \frac{1}{2} & -\frac{1}{2}I \\ 1 & 0 & 0 \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} 1+2\,I & 0 & 0 \\ 0 & 1-2\,I & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

===================================

Number 6

```
> A:=matrix(3,3,[[1,0,0],[-4,1,0],[3,6,2]]);
```

$$A := \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{bmatrix}$$

```
> eigenvals(A);
```

$$1,\ 1,\ 2$$

```
> E:=eigenvects(A);
```

$$E := [2,\ 1,\ \{[0,\ 0,\ 1]\}],\ [1,\ 2,\ \{[0,\ 1,\ -6]\}]$$

A is not diagonizable.

==

Number 7

```
> A:=matrix(3,3,[[4,2,4],[2,1,2],[4,2,4]]);
```

$$A := \begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix}$$

```
> eigenvals(A);
```

$$0,\ 0,\ 9$$

```
> E:=eigenvects(A);
```

$$E := [9,\ 1,\ \{\left[1,\ \frac{1}{2},\ 1\right]\}],\ [0,\ 2,\ \{[0,\ -2,\ 1],\ [1,\ -2,\ 0]\}]$$

```
> S:=transpose(matrix(3,3,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 1 & 0 & 1 \\ \frac{1}{2} & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{4}{9} & \frac{2}{9} & \frac{4}{9} \\ \frac{-4}{9} & \frac{-2}{9} & \frac{5}{9} \\ \frac{5}{9} & \frac{-2}{9} & \frac{-4}{9} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

==================================

Number 8

```
> A:=matrix(2,2,[[3,-4],[1,-1]]);
```

$$A := \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

```
> eigenvals(A);
```

$$1,\ 1$$

```
> E:=eigenvects(A);
```

$$E := [1,\ 2,\ \{[2,\ 1]\}]$$

A is not diagonizable.

==================================

Number 9

```
> A:=matrix(3,3,[[2,2,4],[2,-1,2],[4,2,2]]);
```

$$A := \begin{bmatrix} 2 & 2 & 4 \\ 2 & -1 & 2 \\ 4 & 2 & 2 \end{bmatrix}$$

```
> eigenvals(A);
```

$$7,\ -2,\ -2$$

```
> E:=eigenvects(A);
```

$$E := [7,\ 1,\ \{[2,\ 1,\ 2]\}],\ [-2,\ 2,\ \{[1,\ -2,\ 0],\ [0,\ -2,\ 1]\}]$$

```
> S:=transpose(matrix(3,3,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & -2 \\ 2 & 0 & 1 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{2}{9} & \frac{1}{9} & \frac{2}{9} \\ \frac{5}{9} & \frac{-2}{9} & \frac{-4}{9} \\ \frac{-4}{9} & \frac{-2}{9} & \frac{5}{9} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} 7 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

==

Number 10

```
> A:=matrix(2,2,[[2,5],[-1,-2]]);
```

$$A := \begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}$$

```
> eigenvals(A);
```

$$I,\ -I$$

```
> E:=eigenvects(A);
```

$$E := [I,\ 1,\ \{[-2-I,\ 1]\}],\ [-I,\ 1,\ \{[-2+I,\ 1]\}]$$

```
> S:=transpose(matrix(2,2,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} -2-I & -2+I \\ 1 & 1 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}+I \\ -\frac{1}{2}I & \frac{1}{2}-I \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

==

Example from Linear Algebra supplement

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```
> A:=matrix(2,2,[[1,2],[3,4]]);
```

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

```
> eigenvals(A);
```

$$\frac{5}{2}+\frac{1}{2}\sqrt{33},\ \frac{5}{2}-\frac{1}{2}\sqrt{33}$$

```
> E:=eigenvects(A);
```

$$E := [\frac{5}{2}+\frac{1}{2}\sqrt{33},\ 1,\ \{\left[1,\ \frac{3}{4}+\frac{1}{4}\sqrt{33}\right]\}],\ [\frac{5}{2}-\frac{1}{2}\sqrt{33},\ 1,\ \{\left[1,\ \frac{3}{4}-\frac{1}{4}\sqrt{33}\right]\}]$$

```
> S:=transpose(matrix(2,2,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 1 & 1 \\ \frac{3}{4} + \frac{1}{4}\sqrt{33} & \frac{3}{4} - \frac{1}{4}\sqrt{33} \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{1}{66}(-3+\sqrt{33})\sqrt{33} & \frac{2}{33}\sqrt{33} \\ \frac{1}{66}(3+\sqrt{33})\sqrt{33} & -\frac{2}{33}\sqrt{33} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG :=$$

$$\left[\frac{1}{66}(-3+\sqrt{33})\sqrt{33} + \frac{2}{11}\sqrt{33} + (\frac{1}{33}(-3+\sqrt{33})\sqrt{33} + \frac{8}{33}\sqrt{33})(\frac{3}{4} + \frac{1}{4}\sqrt{33}),\right.$$
$$\left.\frac{1}{66}(-3+\sqrt{33})\sqrt{33} + \frac{2}{11}\sqrt{33} + (\frac{1}{33}(-3+\sqrt{33})\sqrt{33} + \frac{8}{33}\sqrt{33})(\frac{3}{4} - \frac{1}{4}\sqrt{33})\right]$$
$$\left[\frac{1}{66}(3+\sqrt{33})\sqrt{33} - \frac{2}{11}\sqrt{33} + (\frac{1}{33}(3+\sqrt{33})\sqrt{33} - \frac{8}{33}\sqrt{33})(\frac{3}{4} + \frac{1}{4}\sqrt{33}),\right.$$
$$\left.\frac{1}{66}(3+\sqrt{33})\sqrt{33} - \frac{2}{11}\sqrt{33} + (\frac{1}{33}(3+\sqrt{33})\sqrt{33} - \frac{8}{33}\sqrt{33})(\frac{3}{4} - \frac{1}{4}\sqrt{33})\right]$$

```
> DIAG[1,2]:=simplify(DIAG[1,2],'radical');
```

$$DIAG_{1,2} := 0$$

```
> DIAG[2,1]:=simplify(DIAG[2,1],'radical');
```

$$DIAG_{2,1} := 0$$

```
> DIAG[1,1]:=simplify(DIAG[1,1],'radical');
```

$$DIAG_{1,1} := \frac{5}{2} + \frac{1}{2}\sqrt{33}$$

```
> DIAG[2,2]:=simplify(DIAG[2,2],'radical');
```

$$DIAG_{2,2} := \frac{5}{2} - \frac{1}{2}\sqrt{33}$$

```
> evalm(DIAG);
```

$$\begin{bmatrix} \frac{5}{2} + \frac{1}{2}\sqrt{33} & 0 \\ 0 & \frac{5}{2} - \frac{1}{2}\sqrt{33} \end{bmatrix}$$

==

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```
> A:=matrix(2,2,[[1,1],[4,1]]);
```

$$A := \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$$

```
> eigenvals(A);
```

$$-1, 3$$

```
> E:=eigenvects(A);
```

$$E := [3, 1, \{[1, 2]\}], [-1, 1, \{[1, -2]\}]$$

```
> S:=transpose(matrix(2,2,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{-1}{4} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$$

=================================

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```
> A:=matrix(2,2,[[1,-1],[5,-3]]);
```

$$A := \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix}$$

```
> eigenvals(A);
```

$$-1 + I, -1 - I$$

```
> E:=eigenvects(A);
```

$$E := [-1 + I, 1, \{\left[\frac{2}{5} + \frac{1}{5}I, 1\right]\}], [-1 - I, 1, \{\left[\frac{2}{5} - \frac{1}{5}I, 1\right]\}]$$

```
> S:=transpose(matrix(2,2,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} \frac{2}{5}+\frac{1}{5}I & \frac{2}{5}-\frac{1}{5}I \\ 1 & 1 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} -\frac{5}{2}I & \frac{1}{2}+I \\ \frac{5}{2}I & \frac{1}{2}-I \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} -1+I & 0 \\ 0 & -1-I \end{bmatrix}$$

=====================================

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```
> A:=matrix(2,2,[[3,2],[2,5]]);
```

$$A := \begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix}$$

```
> eigenvals(A);
```

$$4+\sqrt{5},\ 4-\sqrt{5}$$

```
> E:=eigenvects(A);
```

$$E := [4+\sqrt{5},\ 1,\ \{\left[1,\ \frac{1}{2}+\frac{1}{2}\sqrt{5}\right]\}],\ [4-\sqrt{5},\ 1,\ \{\left[1,\ \frac{1}{2}-\frac{1}{2}\sqrt{5}\right]\}]$$

```
> S:=transpose(matrix(2,2,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 1 & 1 \\ \frac{1}{2}+\frac{1}{2}\sqrt{5} & \frac{1}{2}-\frac{1}{2}\sqrt{5} \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{1}{10}(\sqrt{5}-1)\sqrt{5} & \frac{1}{5}\sqrt{5} \\ \frac{1}{10}(\sqrt{5}+1)\sqrt{5} & -\frac{1}{5}\sqrt{5} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \left[\frac{3}{10}(\sqrt{5}-1)\sqrt{5}+\frac{2}{5}\sqrt{5}+(\frac{1}{5}(\sqrt{5}-1)\sqrt{5}+\sqrt{5})(\frac{1}{2}+\frac{1}{2}\sqrt{5}), \frac{3}{10}(\sqrt{5}-1)\sqrt{5}+\frac{2}{5}\sqrt{5}+(\frac{1}{5}(\sqrt{5}-1)\sqrt{5}+\sqrt{5})(\frac{1}{2}-\frac{1}{2}\sqrt{5})\right]$$
$$\left[\frac{3}{10}(\sqrt{5}+1)\sqrt{5}-\frac{2}{5}\sqrt{5}+(\frac{1}{5}(\sqrt{5}+1)\sqrt{5}-\sqrt{5})(\frac{1}{2}+\frac{1}{2}\sqrt{5}), \frac{3}{10}(\sqrt{5}+1)\sqrt{5}-\frac{2}{5}\sqrt{5}+(\frac{1}{5}(\sqrt{5}+1)\sqrt{5}-\sqrt{5})(\frac{1}{2}-\frac{1}{2}\sqrt{5})\right]$$

```
> DIAG[1,2]:=simplify(DIAG[1,2],'radical');
```

$$DIAG_{1,\,2} := 0$$

```
> DIAG[2,1]:=simplify(DIAG[2,1],'radical');
```

$$DIAG_{2,\,1} := 0$$

```
> DIAG[1,1]:=simplify(DIAG[1,1],'radical');
```

$$DIAG_{1,\,1} := 4+\sqrt{5}$$

```
> DIAG[2,2]:=simplify(DIAG[2,2],'radical');
```

$$DIAG_{2,\,2} := 4-\sqrt{5}$$

```
> evalm(DIAG);
```

$$\begin{bmatrix} 4+\sqrt{5} & 0 \\ 0 & 4-\sqrt{5} \end{bmatrix}$$

======================================

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```
> A:=matrix(3,3,[[1,2,3],[2,4,5],[3,5,6]]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

```
> eigenvals(A);
```

$$\frac{1}{6}\%1^{1/3}+\frac{266}{3}\frac{1}{\%1^{1/3}}+\frac{11}{3}, -\frac{1}{12}\%1^{1/3}-\frac{133}{3}\frac{1}{\%1^{1/3}}+\frac{11}{3}+\frac{1}{2}I\sqrt{3}(\frac{1}{6}\%1^{1/3}-\frac{266}{3}\frac{1}{\%1^{1/3}}),$$
$$-\frac{1}{12}\%1^{1/3}-\frac{133}{3}\frac{1}{\%1^{1/3}}+\frac{11}{3}-\frac{1}{2}I\sqrt{3}(\frac{1}{6}\%1^{1/3}-\frac{266}{3}\frac{1}{\%1^{1/3}})$$
$$\%1 := 12124+1092\,I\sqrt{3}$$

```
> Digits:=30:  E:=evalf(eigenvects(A));
```

$$E := [11.3448142827620776880944385821, 1., \{[$$
$$.445041867912628808577805128 6 - .1\,10^{-29}\,I,$$
$$.801937735804838252472204639 3 + .1\,10^{-29}\,I,\ 1.]\}], [$$
$$-.515729471589257140261003659053 + .173\,10^{-28}\,I,\ 1., \{[$$
$$-1.24697960371746706105000976796 + .82\,10^{-28}\,I,$$
$$-.554958132087371191422194870949 - .45\,10^{-28}\,I,\ 1.]\}], [$$
$$.170915188827179452166565077013 - .173\,10^{-28}\,I,\ 1., \{[$$
$$1.80193773580483825247220463908 - .73\,10^{-28}\,I,$$
$$-2.24697960371746706105000976796 + .40\,10^{-28}\,I,\ 1.]\}]$$

```
> S:=transpose(matrix(3,3,[op[1](E[1][3]),op[1] (E[2][3]),op[1](E[3][3])]));
```

$$S :=$$
$$[.4450418679126288085778051286 - .1\,10^{-29}\,I\,,$$
$$-1.24697960371746706105000976796 + .82\,10^{-28}\,I\,,$$
$$1.80193773580483825247220463908 - .73\,10^{-28}\,I]$$
$$[.8019377358048382524722046393 + .1\,10^{-29}\,I\,,$$
$$-.554958132087371191422194870949 - .45\,10^{-28}\,I\,,$$
$$-2.24697960371746706105000976796 + .40\,10^{-28}\,I]$$
$$[1.\,,\ 1.\,,\ 1.]$$

```
> T:=inverse(S);
```

$$T :=$$
$$[.241717353090013695661116413858 + .924701916106689405568733042448\,10^{-31}\,I\,,$$
$$.435559619931757901931744915291 - .955591089551671656910553261515\,10^{-31}\,I\,,$$
$$.543133962257834036067457805539 - .158362918171360409833645202061\,10^{-30}\,I]$$
$$[-.435559619931757901931744915322 - .164758694624734042628803732469\,10^{-28}\,I\,,$$
$$-.193842266841744206270628501497 + .473743586280121820533642912856\,10^{-30}\,I\,,$$
$$.349291695416089829796829304093 + .671082150904024476477303479478\,10^{-29}\,I]$$
$$[.193842266841744206270628501464 + .163833992708627353223234999426\,10^{-28}\,I\,,$$
$$-.241717353090013695661116413794 - .378184477324954654842587586705\,10^{-30}\,I\,,$$
$$.107574342326076134135712890365 - .655245859086888435493938959268\,10^{-29}\,I]$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG :=$$
$$[11.3448142827620776880944385820 + .106\,10^{-56}\,I\,,$$
$$.43\,10^{-27} + .261691104351170159003904562 8\,10^{-29}\,I\,,$$
$$.40\,10^{-27} - .249157812529782080181756973 9\,10^{-29}\,I]$$
$$[-.1377\,10^{-26} - .286668992425100982626063230476 6\,10^{-29}\,I\,,$$
$$-.515729471589257140261003659068 + .18\,10^{-57}\,I\,,$$
$$.5\,10^{-29} - .165084218747021791981824302596\,10^{-28}\,I]$$
$$[.16291\,10^{-26} + .486668992425100982626063230467 6\,10^{-29}\,I\,,$$
$$-.33\,10^{-29} + .183830889564882984099609543718\,10^{-28}\,I\,,$$
$$.170915188827179452166565077002 - .90\,10^{-57}\,I]$$

The imaginary parts here are quite small. They are due to roundoff in the calculation. We know for a symmetric matrix we have real eigenvalues and real eigenvectors. So the imaginary parts here are bogus. We can get rid of them by taking the real part.

```
> DIAG:=evalm(Re(DIAG));
```

$$DIAG := \begin{bmatrix} 11.34481428276207768809443858 20\,, .43\,10^{-27}\,, .40\,10^{-27} \\ -.1377\,10^{-26}\,, -.51572947158925714026100365 9068\,, .5\,10^{-29} \\ .16291\,10^{-26}\,, -.33\,10^{-29}\,, .17091518882717945216656507 7002 \end{bmatrix}$$

We still do not have the expected diagonal matrix. The small off-diagonal terms here are in fact also due to roundoff . This can be seen by doing the calculations with more significant digits (for example 60). In this cae the off-diagonal terms will be much smaller.

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```
> A:=matrix(3,3,[[3,2,4],[2,0,2],[4,2,3]]);
```

$$A := \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

```
> eigenvals(A);
```

$$8,\ -1,\ -1$$

```
> E:=eigenvects(A);
```

$$E := [-1,\ 2,\ \{[1,\ -2,\ 0],\ [0,\ -2,\ 1]\}],\ [8,\ 1,\ \{[2,\ 1,\ 2]\}]$$

```
> S:=transpose(matrix(3,3,[op[1](E[1][3]),op[1] (E[2][3])]));
```

$$S := \begin{bmatrix} 0 & 1 & 2 \\ -2 & -2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

```
> T:=inverse(S);
```

$$T := \begin{bmatrix} \frac{-4}{9} & \frac{-2}{9} & \frac{5}{9} \\ \frac{5}{9} & \frac{-2}{9} & \frac{-4}{9} \\ \frac{2}{9} & \frac{1}{9} & \frac{2}{9} \end{bmatrix}$$

```
> DIAG:=evalm(T &* A &* S);
```

$$DIAG := \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$