

Vector Calculus I – Mth 254

Archive – Fall 1994 Files

Sept 27, 1999

This document contains the mid-term and the final exam, from Mth 254 Fall 1994. The original test formatting has not been preserved.

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1 Mid-Term Test

Problem 1. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 3 & -6 & 4 \\ 0 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix}$$

Problem 2. The matrix

$$A = \begin{bmatrix} 6 & -10 & 4 \\ 0 & -3 & 0 \\ 0 & -6 & 3 \end{bmatrix}$$

has eigenvalues

-3 , 3 , and 6 .

Find the corresponding eigenvectors.

Problem 3. The planes

$$\begin{aligned}3x + 4y + 12z &= -90 \\2x + 2y + z &= -54\end{aligned}$$

intersect in a line. Find

(A) The parametric equation of the line of intersection.

(B) $\cos(\theta)$ where θ is the angle between the planes.

Problem 4. Find the arc length of the curve

$$\vec{r}(t) = (2 \cos(t) - 3 \sin(t)) \vec{i} + (3 \cos(t) + 2 \sin(t)) \vec{j} + 6t \vec{k}$$

for $0 \leq t \leq 3$.

Problem 5. Consider the line

$$\vec{r}(t) = (2t + 1) \vec{i} + t^2 \vec{j} + t^3 \vec{k}.$$

(A) Find the parametric equation of the tangent line at $\vec{r}(1)$.

(B) Find the symmetric equations of the tangent line at $\vec{r}(1)$.

Problem 6.

(A). Find all solutions to the linear system

$$\begin{aligned}x - y - 2z &= 1 \\2x + y + z &= 5 \\x + 2y + 3z &= 4\end{aligned}$$

(B). Find all solutions to the linear system

$$\begin{aligned}x - y - 2z &= 2 \\2x + y + z &= 5 \\x + 2y + 3z &= 4\end{aligned}$$

(C). Find all solutions to the linear system

$$\begin{aligned}x - y - z &= 1 \\2x + y + z &= 5 \\x + 2y + 4z &= 4\end{aligned}$$

2 Final Exam

Problem 7. Consider the planes

$$2x + 3y - 6z = 2 \quad \text{and} \quad x - 2y + 2z = 3.$$

1. If θ is the angle between these planes find $\cos(\theta)$.
 2. Find a vector parallel to the line of intersection of these planes.
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Problem 8. The curvature of the graph of $y = f(x)$ is given by

$$\kappa(x) = \frac{|f''(x)|}{(1 + |f'(x)|^2)^{3/2}}.$$

Find the maximum curvature of the graph of

$$y = \frac{3}{\sqrt{5}} x^3$$

for $x \geq 0$.

Problem 9. Let

$$f(x, y) = x^3 - 6xy - 3y^2 - 9x.$$

Find all the critical points of f and use the second derivative test to classify them.

Problem 10. Let

$$f(x, y) = x^2 - 2xy + 2y^2 - x - 2y$$

and let Ω be the triangle with vertices

$$(0, 0), (4, 0) \text{ and } (0, 4).$$

Find the minimum and the maximum of f in Ω .

Problem 11. Let

$$f(x, y) = x^2 - 2xy + 3y^2 - 2x - 5y.$$

1. Find the gradient $\text{grad } f(1, 1)$ of f at $(1, 1)$.
 2. Find the directional derivative of f at $(1, 1)$ in the direction of the vector $3\vec{i} - 4\vec{j}$.
 3. Find the maximum directional derivative of f at $(1, 1)$.
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4. Find the directional derivative of f at $(1, 1)$ in a direction perpendicular to the gradient $\text{grad } f(1, 1)$.
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Problem 12. Suppose the parametric curve $\vec{r}(t)$ has tangent vector

$$\vec{r}'(0) = 2\vec{i} + \vec{j} - 3\vec{k}$$

at $t = 0$. If $f(x, y, z)$ has gradient

$$\text{grad } f(\vec{r}(0)) = 5\vec{i} - 2\vec{j} + 2\vec{k}$$

at $\vec{r}(0)$ and gradient

$$\text{grad } f(\vec{r}(1)) = 4\vec{i} - 3\vec{j} + 2\vec{k}$$

at $\vec{r}(1)$ compute

$$\frac{d}{dt} f(\vec{r}(t))$$

at $t = 0$. Justify your answer.

Problem 13. Use the method of Lagrange multipliers to find the maximum and the minimum of

$$f(x, y) = 5x - 6y - 3 \quad \text{subject to the constraint} \quad \frac{x^2}{4} + \frac{y^2}{16} = 1.$$

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