

Integral Calculus – Mth 252

Archive – Fall 1999 Files

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This archive contains the sample problems and tests from Mth 252 Fall 1999. The original test instructions, headers and formatting have not been preserved.

Contents

1	Sample Test	1
2	Midterm Test	3
3	Make-Up Midterm Test	6
4	Final Exam	9
5	Contact Information	14

1 Sample Test

Problem 1. The antiderivative of $\frac{x + x^2}{1 + x^2}$ is

- A.) $C + \frac{1+2x}{1+x^2} - 2\frac{x^2+x^3}{(1+x^2)^2}$
- B.) $C + \frac{x^2+x^3/3}{x+x^3/3}$
- C.) $C + x + \frac{1}{2}\log(1 + x^2) - \arctan(x)$
- D.) $C + (x^2 + x^3/3)\log(1 + x^2)$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 1).

Problem 2. For $|x| < 1$ find the derivative of

$$x\sqrt{1-x^2} + \arcsin(x).$$

- A.) $\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$
B.) $-\sqrt{1-x^2}$
C.) $2\sqrt{1-x^2}$
D.) $\sqrt{1-x^2}$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 2).

Problem 3. Find the area between the parabola $y = 2x^2 + 1$ and the line $y = 6x + 57$.

- A.) $484/3$
B.) $847/3$
C.) $1331/3$
D.) 444
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 3).

Problem 4. Find the derivative with respect to x of

$$\int_0^{x^2} \arctan(t^3) dt.$$

- A.) $2x \arctan(x^3)$
B.) $2x \arctan(x^6)$
C.) $3x^4/(1+x^6)$
D.) $\arctan(x^6)$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 4).

Problem 5. Use the substitution $u = \arctan(x)$ to evaluate the definite integral

$$\int_0^1 \frac{\arctan(x)}{1+x^2} dx$$

- A.) $\frac{194}{629}$
B.) $\frac{29}{94}$
C.) $\frac{\pi^2}{16}$
D.) $\frac{\pi}{32}$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 5).

Problem 6. If $f(x)$ has the antiderivative (primitive, indefinite integral) $F(x)$ and $F(-1) = 2$, $F(0) = 3$, $F(1) = 5$, $F(2) = 4$, $F(3) = 5$ then

$$\int_0^3 f(x) \, dx =$$

- A.) 0
- B.) 1
- C.) 2
- D.) 3
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 6).

This sample test is not a full length test. Its purpose is to show you what my multiple choice tests look like and also to provide a few sample problems. The real test (midterm) will probably have a few more problems.

2 Midterm Test

Problem 7. The antiderivative of $\frac{1 + \cos(x)}{\cos(x)}$ is

- A.) $C + (1 + \cos(x)) \log(\cos(x))$
- B.) $C + (1 + \tan(x)) \log(\cos(x))$
- C.) $C + x + \log(\sec(x) + \tan(x))$
- D.) $C + x + \log(\cos(x) + \tan(x))$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 7).

Problem 8. If $f(x) = \sec^2(x) \tan(x)$ then $f'(0) = ?$ (The prime denotes the derivative.)

- A.) 0
- B.) 1
- C.) 2
- D.) 3
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 8).

Problem 9. Find the area between the curve $y = \exp(x)$ and the curve $y = \exp(3x)$ over the interval $0 \leq x \leq 1$.

- A.) $\frac{469}{101}$
B.) $\frac{\exp(2)-1}{2}$
C.) $\frac{\exp(3)}{3} - \exp(1)$
D.) $\frac{\exp(3)}{3} - \exp(1) + \log(2)$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 9).

Problem 10. Find the derivative with respect to x of

$$\int_{x^2}^0 \arctan(t^3) dt.$$

- A.) $-2x \arctan(x^3)$
B.) $-2x \arctan(x^6)$
C.) $-3x^4/(1+x^6)$
D.) $-\arctan(x^6)$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 10).

Problem 11. Use the substitution $u = \log(\sec(x))$ to evaluate the integral

$$\int_{\pi/4}^{\pi/3} \frac{\tan(x)}{\log(\sec(x))} dx$$

- A.) $\log(2)$
B.) $\log(\log(2))$
C.) $\frac{61}{88}$
D.) $\frac{2\pi}{9}$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 11).

Problem 12. If $f(x)$ has the antiderivative (primitive, indefinite integral) $F(x)$ and $F(-1) = 2$, $F(0) = 3$, $F(1) = 5$, $F(2) = 4$, $F(3) = 5$ then

$$\int_{-1}^2 f(x) \, dx =$$

- A.) 0
- B.) 1
- C.) 2
- D.) 3
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 12).

Problem 13. For a certain body the plane sections perpendicular to the x -axis are found to have area $x^2 - 1$ where $1 \leq x \leq 2$. Find the volume:

- A.) $\frac{4\pi}{3}$
- B.) $\frac{38\pi}{15}$
- C.) $\frac{2}{3}$
- D.) $\frac{4}{3}$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 13).

Problem 14. A certain body of revolution is “decomposed” into cylindrical shells with axis of symmetry the y -axis. Suppose the shell which has radius x has height $2x^2 + 1$, where $0 \leq x \leq 1$. Find the volume:

- A.) 2π
- B.) $\frac{47}{15}\pi$
- C.) $\frac{5}{3}\pi$
- D.) $\frac{10}{3}\pi$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 14).

Problem 15. Integrate by parts $\int x \log(x) \, dx$.

- A.) $\frac{1}{2}x \log(x) - \frac{1}{4}x^2$
- B.) $\frac{1}{2}x^2 \log(x) - \frac{1}{2}x^2$
- C.) $\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$
- D.) $\frac{1}{2}x^2 \log(x) - \frac{1}{2}x^2$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 15).

Problem 16. Evaluate $\int_1^e x \log(x) dx$ (where $e = \exp(1)$).

- A.) $\frac{151}{72}$
- B.) $\frac{e^2-1}{4}$
- C.) $\frac{e^2}{4}$
- D.) $\frac{e^2+1}{4}$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 16).

3 Make-Up Midterm Test

Problem 17. The antiderivative of $\frac{1 + \sin(x)}{\cos(x)}$ is

- A.) $C + \log |\sec(x) + \tan(x)| + \log |\cos(x)|$
- B.) $C + \log |\sec(x) + \tan(x)| + \log |\sec(x)|$
- C.) $C + x + \log (\sec(x) + \tan(x))$
- D.) $C + x + \log (\cos(x) + \tan(x))$
- E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 17).

Problem 18. $\int_0^{\pi/4} \cos^3(x) dx =$

- A.) $\frac{340}{577}$
- B.) $\frac{2\sqrt{2}}{5}$
- C.) $\frac{5\sqrt{2}}{12}$
- D.) $\frac{3}{16}$
- E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 18).

Problem 19. Find the area between the curve $y = \cos(x)$ and the curve $y = \sin(x)$ over the interval $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$.

- A.) $\frac{82}{29}$
 B.) $2\sqrt{2}$
 C.) $\sqrt{2}$
 D.) 4
 E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 19).

Problem 20. Find the derivative with respect to x of

$$\int_{-x^2}^{x^2} \arctan(t^3) dt.$$

- A.) $2x \arctan(x^3)$
 B.) $4x \arctan(x^6)$
 C.) $6x^4/(1+x^6)$
 D.) 0
 E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 20).

Problem 21. Suppose $0 < a < b < \frac{\pi}{2}$ and suppose $\sec(b) = \sec^3(a)$. Use the substitution $u = \log(\sec(x))$ to evaluate the integral

$$\int_a^b \frac{\tan(x)}{\log(\sec(x))} dx$$

- A.) $\frac{78}{71}$
 B.) $\log(4)$
 C.) $\log(3)$
 D.) $\log(2)$
 E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 21).

Problem 22. If $f(x)$ has the antiderivative (primitive, indefinite integral) $F(x)$ and $F(-1) = 2$, $F(0) = 3$, $F(1) = 5$, $F(2) = 4$, $F(3) = 5$ then

$$\int_0^3 f(x) dx =$$

- A.) 0
- B.) 1
- C.) 2
- D.) 3
- E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 22).

Problem 23. For a certain body the plane sections perpendicular to the x -axis are found to have area $\tan^2(x)$ where $0 \leq x \leq \frac{\pi}{4}$. Find the volume:

- A.) 1
- B.) $4 - \pi$
- C.) $\frac{4-\pi}{4}$
- D.) $\frac{\pi}{4}$
- E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 23).

Problem 24. A certain body of revolution is “decomposed” into cylindrical shells with axis of symmetry the y -axis. Suppose the shell which has radius x has height $\exp(x^2)$, where $0 \leq x \leq 1$. Find the volume:

- A.) $\pi(e - 1)$
- B.) $\frac{e-1}{2}$
- C.) 2π
- D.) πe
- E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 24).

Problem 25. Integrate by parts $\int x \sec(x) \tan(x) dx$.

- A.) $x^2 \sec(x)$
- B.) $x \sec(x)$
- C.) $x \sec(x) + \log |\sec(x)|$
- D.) $x \sec(x) + \log |\tan(x)|$
- E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 25).

Problem 26. Evaluate $\int_0^{2\pi} \cos^4(x) \, dx$.

- A.) $\frac{3}{4}$
 B.) $\frac{3\pi}{4}$
 C.) $\frac{\pi}{2}$
 D.) $\frac{33}{14}$
 E.) None of the above.

← Write the letter corresponding to your answer here. (Problem 26).

Note that $\log(x)$ means the *natural* logarithm of x in this course.

4 Final Exam

Problem 27. Solve the ordinary differential equation

$$\frac{dy}{dx} = \frac{xy}{x^2 + 1}.$$

Simplify your solution so that it contains no logarithms. To obtain full credit your solution must contain an appropriate arbitrary constant.

Problem 28. Use partial fractions to evaluate the indefinite integral

$$\int \frac{2x^2 + 4x + 11}{(x + 1)^2(x - 2)} \, dx.$$

You must show clear work for full credit. Include the arbitrary constant in your solution.

Problem 29. Simplify

$$\exp\left(\frac{1}{2}\log(2) - \log(\sqrt{3} + 1) - \log(\sqrt{3} - 1)\right)$$

where $\log(x)$ is the natural logarithm of x and $\exp(x) = e^x$.

- A.) $\sqrt{2}$
 B.) $-\sqrt{2}$
 C.) $\frac{1}{2}\sqrt{2}$
 D.) $-\frac{1}{2}\sqrt{2}$
 E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 29).

Problem 30. If $\int_5^2 f(t) dt = 4$ and $\int_3^5 f(t) dt = 6$ then

$$\int_2^3 f(t) dt =$$

- A.) 2
 B.) -2
 C.) 10
 D.) -10
 E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 30).

Problem 31. If we substitute $u = \sin(x)$ in the integral

$$\int \cos^4(x) \sin^6(x) dx$$

we obtain

- A.) $\int (1 - u^2)^{3/2} u^6 du$
 B.) $\int (1 - u^2)^2 u^6 du$
 C.) $\int (1 - u^2)^3 u^6 du$
 D.) $\int (1 - u^2)^{5/2} u^6 du$
 E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 31).

Problem 32. If we substitute $u = \sin(x)$ in the integral

$$\int \cos^5(x) \sin^6(x) dx$$

we obtain

- A.) $\int (1 - u^2)^{3/2} u^6 du$
 B.) $\int (1 - u^2)^2 u^6 du$
 C.) $\int (1 - u^2)^3 u^6 du$
 D.) $\int (1 - u^2)^{5/2} u^6 du$
 E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 32).

Problem 33. Find the area of the bounded region bounded by the parabolas

$$y = x^2 - 4x + 4 \quad \text{and} \quad y = -x^2 + 6x - 8.$$

- A.) 1
- B.) $\frac{1}{2}$
- C.) 3
- D.) $\frac{1}{3}$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 33).

Problem 34. The area under the graph of $y = -x^2 + 3x - 2$, $1 \leq x \leq 2$ is rotated about the y -axis. Find the volume swept out.

- A.) $\frac{\pi}{4}$
- B.) $\frac{\pi}{2}$
- C.) $\frac{\pi}{6}$
- D.) $\frac{\pi}{30}$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 34).

Problem 35. The area under the graph of $y = -x^2 + 3x - 2$, $1 \leq x \leq 2$ is rotated about the x -axis. Find the volume swept out.

- A.) $\frac{\pi}{4}$
- B.) $\frac{\pi}{2}$
- C.) $\frac{\pi}{6}$
- D.) $\frac{\pi}{30}$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 35).

Problem 36. For a certain solid the plane sections perpendicular (normal, orthogonal) to the x -axis are equilateral triangles with edge length $(x^2 + 1)^{1/2}$, $0 \leq x \leq 2$. Find the volume.

- A.) $\frac{14}{3}$
B.) $\frac{7}{3}$
C.) $\frac{7}{3}\sqrt{3}$
D.) $\frac{7}{6}\sqrt{3}$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 36).

Problem 37. Find the length of the graph $y = 2x^{3/2}$ for $0 \leq x \leq \frac{5}{3}$.

- A.) 42
B.) $\frac{128}{27}$
C.) $\frac{14}{3}$
D.) $\frac{128}{3}$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 37).

Problem 38. If we integrate by parts $\int (2 + \log(x))^2 dx$ by setting $u = (2 + \log(x))^2$ and $dv = dx$ we obtain

- A.) $(2 + \log(x))^2 x - 2 \int (2 + \log(x)) dx$
B.) $(2 + \log(x))^2 x - \int (2 + \log(x)) dx$
C.) $(2 + \log(x))^2 - 2 \int (2 + \log(x)) dx$
D.) $(2 + \log(x))^2 - \int (2 + \log(x)) dx$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 38).

Problem 39. If we integrate by parts $\int (1 + x^2) \cos(x) dx$ by setting $u = 1 + x^2$ and $dv = \cos(x) dx$ we obtain

- A.) $(1 + x^2) \cos(x) + 2 \int x \cos(x) dx$
B.) $(1 + x^2) \cos(x) - 2 \int x \cos(x) dx$
C.) $(1 + x^2) \sin(x) + 2 \int x \sin(x) dx$
D.) $(1 + x^2) \sin(x) - 2 \int x \sin(x) dx$
E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 39).

Problem 40. Integrate by parts $\int x \sec^2(x) dx$.

- A.) $C + \tan(x) - \log |\cos(x)|$
 B.) $C + \tan(x) + \log |\cos(x)|$
 C.) $C + x \tan(x) - \log |\cos(x)|$
 D.) $C + x \tan(x) + \log |\cos(x)|$
 E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 40).

Problem 41. Substitute $u = \sec(x)$ to evaluate the integral

$$\int \sec^6(x) \tan(x) dx.$$

- A.) $C + \frac{\sec^6(x)}{6}$
 B.) $C + \frac{\sec^7(x)}{7}$
 C.) $C + \frac{\sec^6(x) \tan(x)}{6}$
 D.) $C + \frac{\sec^7(x) \tan(x)}{7}$
 E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 41).

Problem 42. Substitute $u = 1 + x^2$ in the integral

$$\int (1 + x^2)^{1/3} dx.$$

Do not evaluate the integral.

- A.) $\frac{1}{2} \int u^{1/3} du$
 B.) $\frac{1}{2} \int u^{1/3}/(u-1) du$
 C.) $\frac{1}{2} \int u^{1/3}/\sqrt{u-1} du$
 D.) $\int u^{1/3}/(u-1) du$
 E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 42).

Problem 43. If $p(x)$ is a polynomial of degree ≤ 3 and the partial fraction expansion of

$$\frac{p(x)}{(x+2)(x^2+4)(x+4)} \quad \text{is} \quad \frac{3}{x+2} + \frac{x+2}{x^2+4} - \frac{1}{x+4}$$

then find $p(x)$.

- A.) $4x^3 + 18x^2 + 28x + 56$
- B.) $3x^3 + 18x^2 + 28x + 56$
- C.) $2x^3 + 18x^2 + 28x + 56$
- D.) $x^3 + 18x^2 + 28x + 56$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 43).

Problem 44. Select the substitution recommended by the methods of this course for evaluating the integral

$$\int \frac{x^3}{(9x^2 - 4)^{3/2}} dx.$$

Do not evaluate the integral.

- A.) $3x = 2 \tan \theta$
- B.) $3x = 2 \sin \theta$
- C.) $3x = 2 \sec \theta$
- D.) $3x = 2 \log \theta$
- E.) None of the above.

← Write letter corresponding to your answer here and mark it on the scantron (Problem 44).

5 Contact Information

The contact information below is accurate as of Feb 22, 2001.

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