Numerical Harmonic Analysis (MTH 656)

Homework due Monday, May 19, 2008.

Problem 11. Research applications of the continuous wavelet transform, for example on the internet. List at least three applications. Write a descriptive paragraph about one of them, including an explanation why the continuous wavelet transform is useful for this application. What is your impression of the kind of application where this transform is most useful?

(20 points)

Problem 12. Show that if $\psi \in L_1 \cap L_2$ and $f \in L_\infty \cap L_2$, then

$$\lim_{a \to 0} T_\psi f(a, b) = 0.$$

(10 points)

Problem 13. Consider the signal

$$f(x) = \begin{cases} 1 & -1/2 \leq x \leq 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

a) Using a Haar wavelet centered at the origin

$$\psi(x) = \begin{cases} 1 & -1/2 \leq x \leq 0 \\ -1 & 0 \leq x \leq 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

do the following:

i) Evaluate $T_\psi f(a, b)$ for $a = 1/2, 1, 2$ and all shifts $b \in \mathbf{R}$.

ii) Sketch $T_\psi f(a, b)$ for all $a > 0$ and b and indicate special behavior, if any (for example regions where $T_\psi f(a, b)$ is zero, behavior as $a \to 0$, anything else of interest.

b) Repeat part a) using the Gaussian wavelet $\psi(x) = xe^{-x^2}$ and compare your observations with those of part a).

(30 points)