This is a 50-minute closed-book test. Show work. Use backs of pages if necessary.

A. Suppose $y_1, y_2, ..., y_n$ are independent Bernoulli random variables, $B(1, \pi_i)$, with $\text{logit}(\pi_i) = \beta x_i$, for scalar parameter $\beta$ and known explanatory variable $x_i$.

1. Write the inverse of the logit function for this model. That is, write $\pi_i$ as a function of $\beta x_i$.

2. According to this model, what is the value of $x$ at which the probability is .5?

3. Describe how much more the odds (of $y$ being 1) are at $x = 7$ than at $x = 5$.

4. Write the log-likelihood function of $\beta$.

5. Suppose the deviance for the model above is 39 (on 33 d.f.) and the deviance for the model that also includes an intercept is 30 (on 32 d.f.). What is the likelihood ratio test statistic for the hypothesis that the intercept is zero?
B. Suppose observations $y_1, y_2, \ldots, y_{10}$ are available from a distribution $f(y_i|x_i; \beta)$ with scalar parameter $\beta$ and log-likelihood function

$$l(\beta) = \beta \sum_{i=1}^{10} y_i x_i - \frac{1}{2} \beta^2 \sum_{i=1}^{10} x_i^2.$$ 

Suppose $\sum_{i=1}^{10} y_i x_i = 110$ and $\sum_{i=1}^{10} x_i^2 = 15$.

1. What is the numerical value of the Wald test statistic for $H_0: \beta = 6$? (Show work.)

2. What is the numerical value of the likelihood ratio test statistic for $H_0: \beta = 6$? (Show work.)