MTH 311/Peszynska Midterm, 10/31/2003

Name

Student ID #

# code name

Instructions:

• Show enough work to justify your answer.

• Use provided space for questions 1-8, in particular, put your answer in a box when provided.

• For problems 9-12, use your own blank sheets and mark which three of the four problems you are solving on the exam sheets.

• No notes or books are allowed.

Write neatly!

GOOD LUCK
1. (1 pts) What is Binomial Formula? (formulate)

2. (1 pts) Is \( \mathbb{N} \) dense in \( \mathbb{R} \)? Explain.

3. (2 pts) What is an injection (Hint: 1-1 function)? Complete the sentence:
   \[ g : X \rightarrow Y \text{ is an injection (1-1 function) if} \]

   Construct an example (if possible) of a function \( g : \mathbb{R} \rightarrow \{0, 1\} \) which is injective (1-1).

4. (1 pts) Define a convergent sequence.

5. (1 pts) Define a bounded sequence.
6. (4 pts) Give an example, if possible, of

• a sequence with three convergent subsequences,

• a monotone sequence which is unbounded,

• a sequence which is not convergent,

• a sequence which is convergent and unbounded.

7. (1 pts) Find the least upper bound (sup) and the largest lower bound (inf) of the set

\[ E := \{ p/q \in \mathbb{Q} : p^2 < 2q^2 \text{ and } p, q > 0 \} \]

\[ \sup E = \quad \inf E = \]

8. (4 pts) Find the limit (if it exists) of the sequence or of its convergent subsequences, if they exist (allow infinite limits), and determine whether the sequence is i) monotone, ii) bounded, for the following sequences:

\[ (-1)^n, 1 + \frac{1}{n}, (-1)^{3n} \]

<table>
<thead>
<tr>
<th>sequence</th>
<th>limit?</th>
<th>convergent subsequence(s)?</th>
<th>monotone?</th>
<th>bounded?</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1)^n)</td>
<td>(-1)</td>
<td>({-1, 1})</td>
<td>(\text{No})</td>
<td>(\text{Yes})</td>
</tr>
<tr>
<td>(1 + \frac{1}{n})</td>
<td>(\infty)</td>
<td>(--)</td>
<td>(\text{No})</td>
<td>(\text{Yes})</td>
</tr>
<tr>
<td>((-1)^{3n})</td>
<td>({1})</td>
<td>(--)</td>
<td>(\text{No})</td>
<td>(\text{Yes})</td>
</tr>
</tbody>
</table>
Pick any three of the following four. Do not work on more than three problems! Each problem is worth 5pts. Mark which problem you choose in the box provided.

9. (________ pts) Prove that $|x| \leq 1$ implies $|x^2 - 1| \leq 2|x - 1|$.

10. (________ pts) Prove that $0 \leq a < b$ implies that for all $n \in \mathbb{N}$,

   $0 \leq a^n < b^n$.

11. (________ pts) Assume $\lim_{n \to \infty} x_n = a$ and $\lim_{n \to \infty} y_n = b$.

What can you say about the limit of the sequence $u_n = x_n - y_n$? Prove it.

12. (________ pts) Discuss convergence of a geometric sequence.