MTH 311/Peszynska, Problems Type II, set 2, discussed 10/17

1. Exercises from the textbook: Note: Numbers like 1.1/7/p.11 refer to the exercise #7 listed after Section 1.1 of the textbook on p.11.
   - 2.1/4,8/p.37
   - 2.3/2,11/p.47
   - 2.4/6,7/p.51

2. Find the limit (if it exists) of the sequence or of its convergent subsequences, if they exist (allow infinite limits):

   \[ n, n^2, \sqrt{n}, \log n \]

   \[ \frac{1}{n+1}, \frac{n}{2n+1}, \frac{3n}{\sqrt{2n^2+1}}, \frac{1}{\log n}, \frac{1}{\sqrt{n}}, \frac{2n^2+1}{\sqrt{n}}, \frac{2n+1}{\sqrt{n+1}} \]

   \[ \cos(n), \cos\left(\frac{\pi}{2}n\right), \cos(\pi n), \cos(2\pi n) \]

   \[ (-1)^n, (-1)^{2n}, (-1)^{2n+1}, (-1)^{3n} \]

   \[ \frac{(-1)^n}{n}, (-1)^n n \]

   \[ (1 + \frac{1}{n}), (1 + \frac{1}{n})^n \]

   \[ \frac{(n!)^2}{(n^2)!} \]

3. Discuss the properties of the sequences listed above: for each sequence determine if it is i) monotone, ii) bounded, iii) Cauchy. You can use all known facts (Theorems, Lemmas) as well as what you already know about this sequence’s convergence.

4. Is the sequence of harmonic numbers \( H_n = 1 + 1/2 + \cdots + 1/n \) convergent?