1. Exercises from the textbook: **Note:** Numbers like 1.1/7/p.11 refer to the exercise #7 listed after Section 1.1 of the textbook on p.11.
   - 3.1/3,5,8,9
   - 3.2/2,3
   - 3.3/7,8

2. Discuss limits and continuity of the Dirichlet function $D(x)$, the Entier function $E(x) := [x] = [x]$, the “signum” function $\text{sgn}(x)$ and the Heaviside function $H(x)$.

   Note: these are defined as:
   $$D(x) := \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^C \end{cases}, \quad E(x) := \sup\{k \in \mathbb{Z} : k \leq x\}$$
   $$\text{sgn}(x) := \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}, \quad H(x) := \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

3. Suppose $g : \mathbb{R} \mapsto \mathbb{R}$ is continuous and that $g(x) = 0$ for every $x \in \mathbb{Q}$. What can you say about $g(x), x \in \mathbb{Q}^C$? Prove it.

4. Define
   $$g(x) = \begin{cases} x^2 & ,x \in \mathbb{Q} \\ -x^2 & ,x \in \mathbb{Q}^C \end{cases}$$

   At what points is $g : \mathbb{R} \mapsto \mathbb{R}$ continuous?

5. Let $f(x) : 4 - 2x^3$ be defined on $[-1, 1]$.
   a) Is the maximum and minimum of $f$ attained on i) $(-1, 1)$, ii) $[-1, 1]$ ? (answer this without calculating).
   b) Find the points at which the maximum is attained.

6. Let $f : [0, \infty) \mapsto \mathbb{R}$ be a continuous function such that $f(0) = 0$ and $f(x) \geq \sqrt{x}$ for all $x \geq 0$. Show that for each $c > 0$ there is some $x > 0$ such that $f(x) = c$.

7. For a function $f : D \mapsto \mathbb{R}$, a solution of the equation $f(x) = x, x \in D$ is called a fixed point of $f : D \mapsto \mathbb{R}$.
   a) If $f(-1) > -1$, and $f(1) < 1$, show that $f : [-1, 1] \mapsto \mathbb{R}$ has a fixed point.
   b) Let $D = \mathbb{R}$ and let $f(\mathbb{R})$ be bounded. Show that there is a solution of the equation $f(x) = x, x \in \mathbb{R}$.