Projectile Motion

Objectives:

- Understand projectile motion and how it is influenced by gravity
- Understand the effects of projection speed, angle, and relative height on projectile motion
- Learn to compute the maximum height, flight time, and flight distance of a projectile

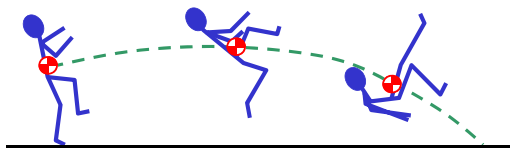
Questions to Think About

- If you want to throw a ball as far as possible, at what angle should you release it?
- What can a diver do to increase the peak height of his dive?
- How can a gymnast increase her time in the air during a dismount?
- How can a punter increase both the distance *and* the hang time of his punts?
- To shoot a basketball accurately, should you use a high arc or a low arc?

What is a Projectile?

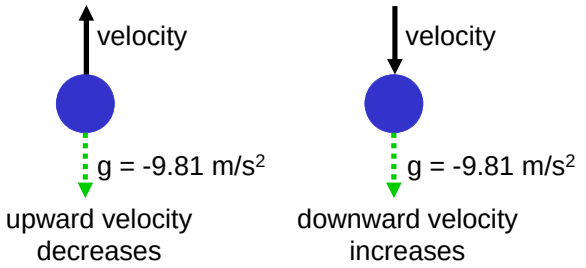
- A projectile is a body or object that:
  - Is in the air
  - Is subject only to the forces of gravity and air resistance

  (*i.e.* the object is in free fall)
- The motion of the center of mass of any object in free fall is governed by the laws of projectile motion



Influence of Gravity

- Gravity: pull of the mass of the Earth on a body
- Accelerates an object towards Earth's center
- Acceleration due to gravity (g) is straight down, can usually assume = 9.81 m/s^2 (= 32.2 ft/s^2)
- Actually depends on distance from Earth

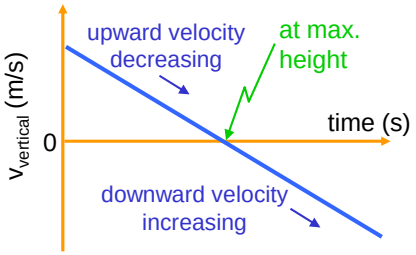


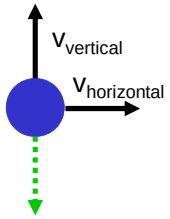
Gravity & Vertical Velocity

- From the laws of constant acceleration:

$$(v_{vertical})_1 = (v_{vertical})_0 + (-9.81\ m/s^2)\ \Delta t$$

- Vertical velocity changes linearly with time





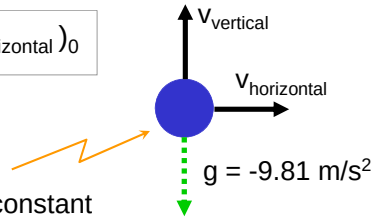
Gravity & Horizontal Velocity

- Gravity does not change the horizontal velocity of an object
- From the laws of constant acceleration:

$$(v_{horizontal})_1 = (v_{horizontal})_0 + a_{horizontal}\ \Delta t$$

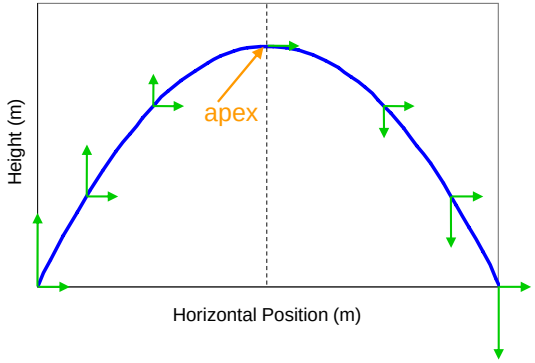
For gravity, $a_{horizontal} = 0$, so:

$$(v_{horizontal})_1 = (v_{horizontal})_0$$



Projectile Motion

- Gravity causes a projectile to move in a parabolic path that is symmetric about the apex (the highest point in the trajectory)

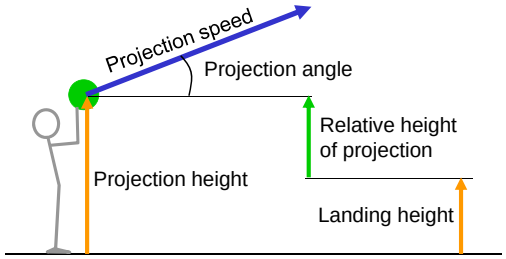


Why is Projectile Motion Important?

- Success in many sports involves projectile motion
- Objects Acting as Projectiles:
  basketball, baseball, football, shot put, hammer, discus, javelin, golf, volleyball, tennis, archery
- The Body Acting as a Projectile:
  high jump, long jump, gymnastics, figure skating, diving, ski jumping
- Variables of interest include:
  - Flight distance
  - Flight time
  - Maximum height

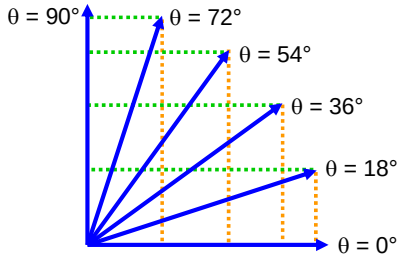
Influences on Projectile Trajectory

- Three factors that influence projectile trajectory:
  - Projection angle
  - Projection speed
  - Relative height of projection
    = (projection height) – (landing height)



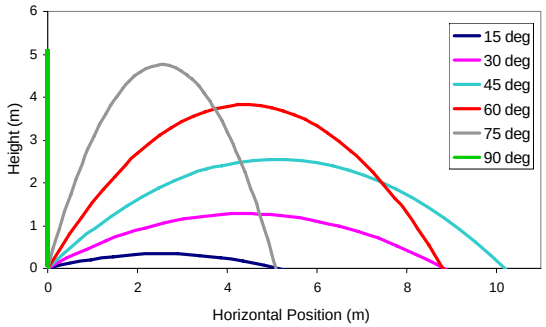
Effect of Angle on Velocity Components

- Trade-off exists between horizontal and vertical projection velocity
- If v_0 = projection speed; θ = projection angle:
  - Horizontal projection velocity = $v_0 \cos \theta$
  - Vertical projection velocity = $v_0 \sin \theta$



Influences of Projection Angle

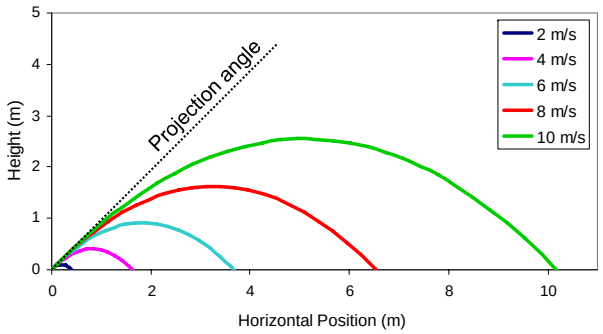
- Effect of projection angle on object trajectory
  (projection speed = 10 m/s, projection height = 0)



- Trajectory shape depends only on projection angle

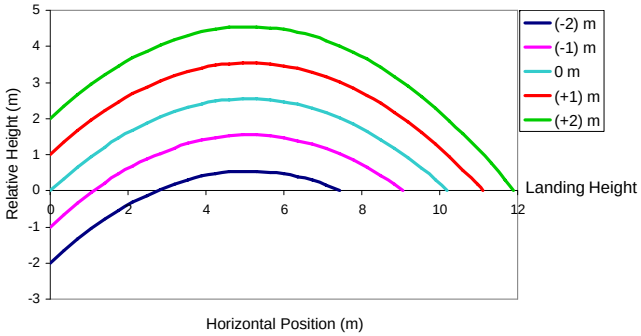
Influences of Projection Speed

- Effect of projection speed on object trajectory
  (projection angle = 45°, projection height = 0)



Influences of Projection Height

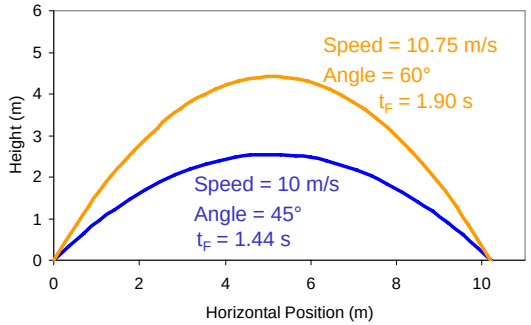
- Effect of relative projection height on object trajectory (projection speed = 10 m/s, projection angle = 45°)



Relative projection ht. = (projection ht.) – (landing ht.)

Trade-off Between Factors

- Can obtain the same flight distance, height, or time with different combinations of projection speed, angle, and height

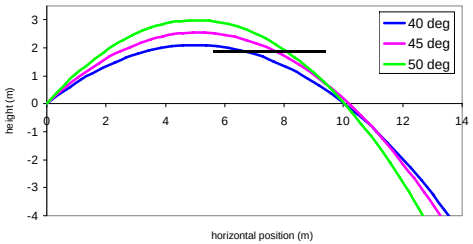


Effects on Projectile Motion

Variable	Determined by:
Horizontal velocity ⬆	Projection speed Projection angle
Vertical velocity ⬆	Projection speed Projection angle
Maximum height ⬆	Vertical velocity Projection height
Flight time ⬆	Vertical velocity Projection height Final height
Flight distance ⬆	Horizontal velocity Flight time

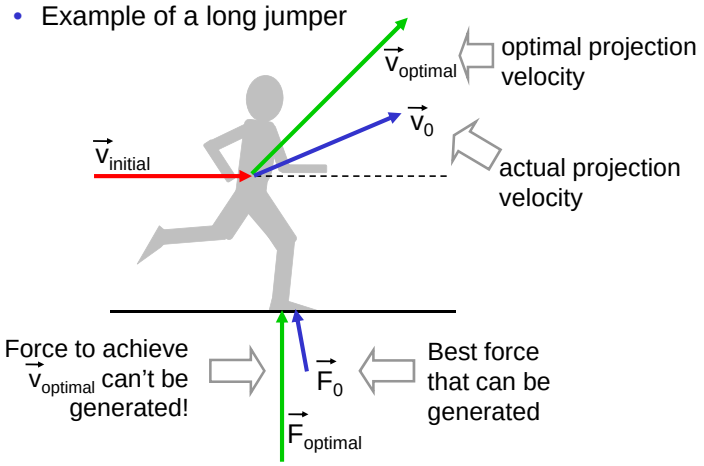
Optimum Projection Angle

- Projection angle for maximum distance depends on relative projection height and projection speed
  - Rel. projection ht. > 0 ➡ Optimal angle < 45°
  - Rel. projection ht. = 0 ➡ Optimal angle = 45°
  - Rel. projection ht. < 0 ➡ Optimal angle > 45°
  - For greater projection speed, optimal angle closer to 45°
- Projection angle for maximum height = 90°



Optimal vs. Actual Conditions

- Example of a long jumper

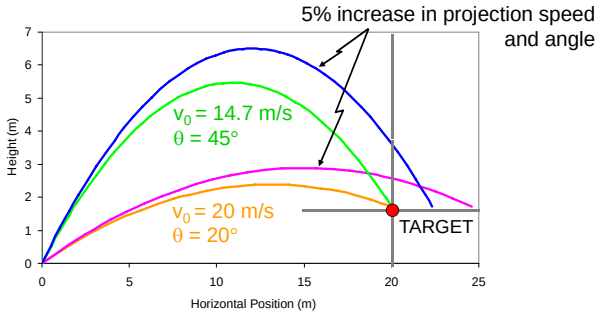


Actual Projection Conditions

- In real life, often cannot attain theoretical optimum conditions
- Trade-off between projection speed, angle, and height due to anatomical & physiological constraints

Projecting for Accuracy

- Vertical-Plane Targets (e.g. baseball pitch, archery)
  - Flatter trajectory provides greater accuracy
- Horizontal-Plane Targets (e.g. basketball, golf)
  - Steeper trajectories provide greater accuracy



Maximum Height

- At the apex, $v_{vertical} = 0$
- From the laws of constant acceleration:

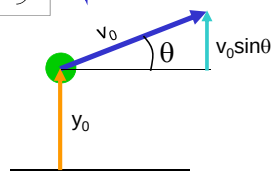
$$v_{yf}^2 = v_{yi}^2 + 2\, a_y\, d_y$$

$$0 = (v_0 \sin\theta)^2 + 2\,(-9.81\ m/s^2)\,(y_{apex} - y_0)$$

$$y_{apex} = y_0 + \frac{(v_0 \sin\theta)^2}{2\,(9.81\ m/s^2)}$$

where:

y_{apex}	= height at apex
y_0	= projection height
v_0	= projection speed
θ	= projection angle



Example Problem #1

A high jumper leaves the ground with a velocity of 6 m/s at a projection angle of 40°. Her center of mass is 1 m above the ground at take-off

What is the maximum height of her center of mass during the jump?

Flight Time

- From the laws of constant acceleration:

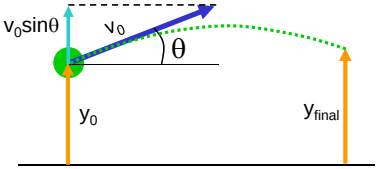
$$d_y = v_{yi}\,\Delta t + (\tfrac{1}{2})\,a_y\,(\Delta t)^2$$

$$(y_{final} - y_0) = (v_0 \sin\theta)\,t_F + (\tfrac{1}{2})\,(-9.81\ m/s^2)\,t_F^2$$

Solve the above quadratic equation to find the flight time t_F (choose the largest positive answer)

where:

y_{final}	= final height
y_0	= projection height
v_0	= projection speed
θ	= projection angle



Quadratic Equations

Quadratic equation:

$$ax^2 + bx + c = 0$$

A quadratic equation has two solutions:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Example Problem #2

A figure skater is attempting a jump in which she performs 3 revolutions while in the air

She leaves the ice with a velocity of 7 m/s at a projection angle of 30°

How long will she be in the air?

If she spins at 3 revolutions per second, will she be able to complete all 3 revolutions before landing?

Flight Distance

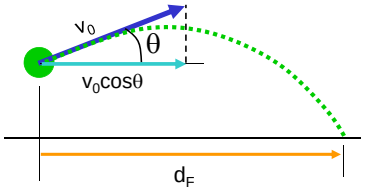
- During projectile motion, $v_{horizontal} = v_0 \cos\theta$ is constant
- From the laws of constant acceleration with $a_x = 0$:

$$d_x = v_{xi}\,\Delta t$$

$$d_F = (v_0 \cos\theta)\, t_F$$

where:

d_F	= flight distance
t_F	= flight time
v_0	= projection speed
θ	= projection angle



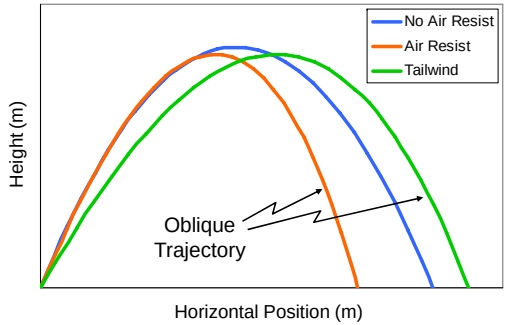
Example Problem #3

A golfer strikes the ball so that it leaves the ground with an initial velocity of 75 m/s at a projection angle of 10°

If the fairway is level, how far away will the ball hit the ground?

Influence of Air Resistance

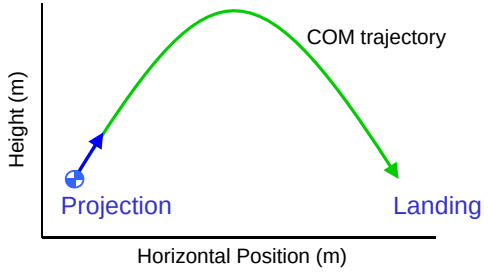
- In real life, air resistance causes both horizontal and vertical velocity to change while in flight.
- Forces created by wind will also affect the trajectory



Projectiles and Performance

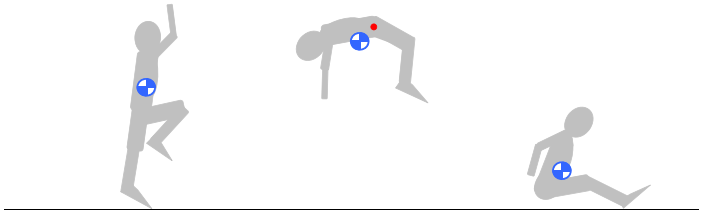
- If we neglect air resistance:

  Trajectory of the center of mass throughout flight is entirely determined at time of projection
- Conditions at time of projection critical to performance where projectile motion involved



Influences of Body Position

- Can use changes in body position to:
  - Increase take-off height of COM (e.g. raising arms)
  - Decrease landing height (e.g. lifting legs)
  - Increase height of individual body parts during flight (by lowering other parts)
  - Increase forward position of individual body parts during flight (by moving other parts backwards)



Example: High Jump

- Trajectory of the center of mass during flight is entirely determined at time jumper leaves ground
  - Includes maximum height of center of mass
- Jumper uses changes in body position in midair to improve performance