Solving Vector Problems

Objectives:

- Define the concept of a vector
- Learn how to perform basic vector operations using graphical and numerical methods
- Learn how to use vector algebra to solve simple problems

Why Vectors?

Many important quantities in biomechanics are meaningless without knowing both a magnitude and a direction:

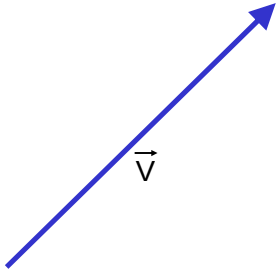
- Position
- Change in position (Displacement)
- Rate of change of position (Velocity)
- Rate of change of velocity (Acceleration)
- Applied forces

Need a way of describing these quantities

Vector

A **vector** is a quantity that has:

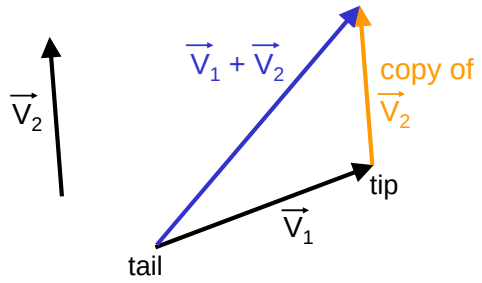
- Magnitude
- Direction



A **scalar** quantity has magnitude only (*e.g.* time)

Graphical Vector Addition

- Use *tip-to-tail* method:



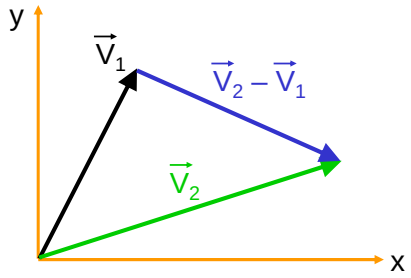
resultant : vector resulting from the math operation

Graphical Vector Subtraction

- Subtraction gives *difference* or *change* between two vectors that start from the same point

$\vec{V}_2 - \vec{V}_1$ = change from $\vec{V}_1$ to $\vec{V}_2$

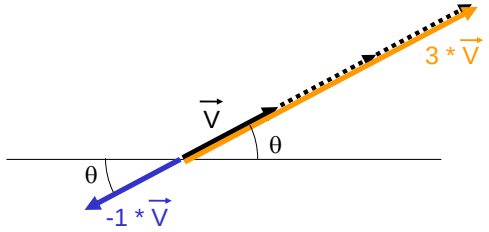
= vector from tip of $\vec{V}_1$ to tip of $\vec{V}_2$



Graphical Vector-Scalar Multiplication

If a vector $\vec{V}$ is multiplied by a scalar *n*:

- Resultant vector is |*n*| times as long
- If *n* > 0: resultant in same direction as $\vec{V}$
- If *n* < 0: resultant in opposite direction to $\vec{V}$



Graphical Solution Using Vectors

1. Establish a scaling factor for the graph (*e.g.* 1cm = 10 m/s)
2. Carefully draw vectors with the correct length (based on the scaling factor) and direction
3. Use graphical addition, subtraction, and/or scalar multiplication to find desired resultant
4. Carefully measure the length and direction of the resultant
5. Use the scaling factor to convert the measured length to the corresponding magnitude

Example Problem #1

Two volleyball players simultaneously contact the ball above the net.

Player #1 hits the ball from the left with a force of 200 N (45 lb), angled 45° below the horizontal.

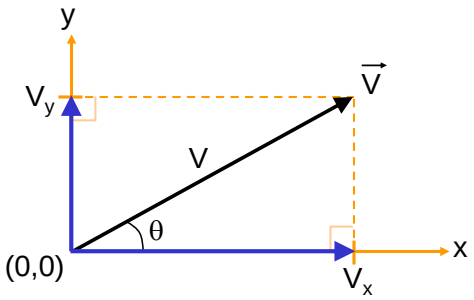
Player #2 hits the ball from the right with a force of 180 N (40 lb), angled 15° below the horizontal.

What is the magnitude and direction of the net (total) force applied to the ball by the two players?

Numerical Representation

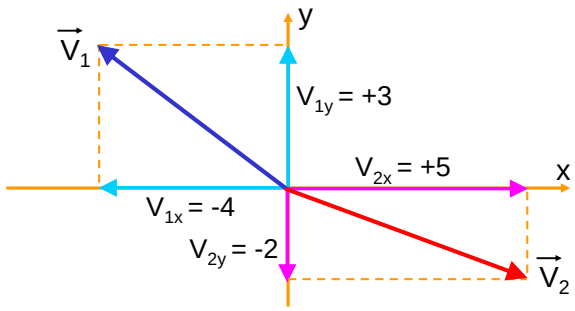
Methods of expressing a vector ($\vec{V}$) numerically:

- Magnitude (V) and direction (θ) with respect to a reference axis
- Components (V_x, V_y) along each reference axis



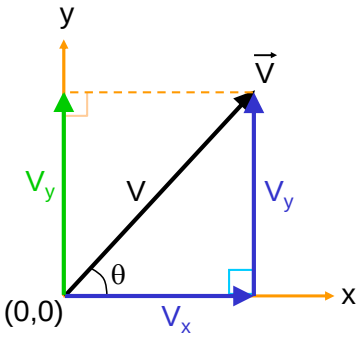
Positive & Negative Components

- If a component points in the positive direction, its value is positive
- If a component points in the negative direction, its value is negative



Resolution into Components

- Trigonometry used to separate a vector into x and y components
- Add a minus sign if component is negative



$$\cos\theta = \frac{V_x}{V}$$

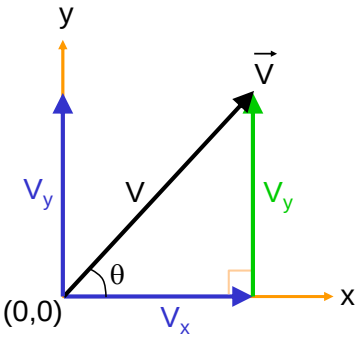
$$\sin\theta = \frac{V_y}{V}$$

$$V_x = V * \cos\theta$$

$$V_y = V * \sin\theta$$

Composition of Components

- A vector can be numerically composed from its components using geometry and trigonometry



$$V^2 = V_x^2 + V_y^2$$

$$\tan\theta = \frac{V_y}{V_x}$$

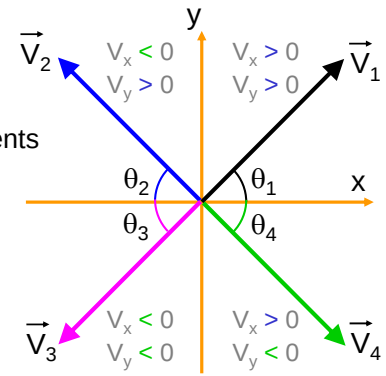
$$V = \sqrt{V_x^2 + V_y^2}$$

$$\theta = \text{atan}\left(\frac{V_y}{V_x}\right)$$

Finding the Correct Angle

- When x or y component is negative, use absolute (positive) values in atan() function:

$$\theta = \text{atan}\left(\frac{|V_y|}{|V_x|}\right)$$



- Signs of x, y components give direction:
  - θ_1 above +x axis
  - θ_2 above -x axis
  - θ_3 below -x axis
  - θ_4 below +x axis

Vector Math

- Perform ordinary math on the x and y components
- Addition:

$$\vec{V}_R = \vec{V}_1 + \vec{V}_2 \quad \Leftrightarrow \quad \begin{aligned} V_{Rx} &= V_{1x} + V_{2x} \\ V_{Ry} &= V_{1y} + V_{2y} \end{aligned}$$

- Subtraction:

$$\vec{V}_R = \vec{V}_1 - \vec{V}_2 \quad \Leftrightarrow \quad \begin{aligned} V_{Rx} &= V_{1x} - V_{2x} \\ V_{Ry} &= V_{1y} - V_{2y} \end{aligned}$$

- Multiplication by a scalar *n*:

$$\vec{V}_R = n * \vec{V}_1 \quad \Leftrightarrow \quad \begin{aligned} V_{Rx} &= n * V_{1x} \\ V_{Ry} &= n * V_{1y} \end{aligned}$$

- Positive resultant points in the + direction
- Negative resultant points in the – direction

Vector Algebra

If:

$$\vec{V}_1 = \vec{V}_2 \quad \Leftrightarrow \quad \begin{aligned} V_{1x} &= V_{2x} \\ V_{1y} &= V_{2y} \end{aligned}$$

Then:

$$\vec{V}_1 + \vec{V}_3 = \vec{V}_2 + \vec{V}_3 \quad \Leftrightarrow \quad \begin{aligned} V_{1x} + V_{3x} &= V_{2x} + V_{3x} \\ V_{1y} + V_{3y} &= V_{2y} + V_{3y} \end{aligned}$$

$$\vec{V}_1 - \vec{V}_3 = \vec{V}_2 - \vec{V}_3 \quad \Leftrightarrow \quad \begin{aligned} V_{1x} - V_{3x} &= V_{2x} - V_{3x} \\ V_{1y} - V_{3y} &= V_{2y} - V_{3y} \end{aligned}$$

$$n * \vec{V}_1 = n * \vec{V}_2 \quad \Leftrightarrow \quad \begin{aligned} n * V_{1x} &= n * V_{2x} \\ n * V_{1y} &= n * V_{2y} \end{aligned}$$

where $\vec{V}_3$ is any vector and *n* is any scalar

Note: can't multiply or divide a vector by a vector

Numerical Solutions Using Vectors

1. Sketch the vectors on a diagram of the problem
2. Choose and draw x and y axes
3. Label known vector magnitudes and angles
4. Find the x and y components of each vector
5. Use algebra to find x component of result from x components of vectors
6. Use algebra to find y component of result from y components of vectors
7. Find the magnitude and angle of the result from its components

Example Problem #2

Find the numerical solution to Example Problem #1

Example Problem #3

A soccer ball is 4 m away and 70° to your left

Your opponent is 2 m away from you and 140° to your left

How far and in what direction does your opponent need to run in order to get to the ball?

Graphical vs. Numerical Method

- Graphical Method
  - Simple and fast
  - Must be done by hand
  - Gives approximate result
- Numerical Method
  - Requires complex calculations
  - Gives accurate result
  - Can be performed by computer
  - Can perform analyses in 3 dimensions
- **On homework and exams, use numerical method unless told otherwise**