Lesson 13

Nuclear Astrophysics
Elemental and Isotopic Abundances
Elemental and Isotopic Abundances (cont.)
Elemental and Isotopic Abundances
Overview of the Sun and the Nucleosynthetic Processes Involved
Primordial Nucleosynthesis

- Age of the universe 10-20 billion years with best estimate being $14 \pm 1 \times 10^9$ y.
- Universe started with the Big Bang.
- Evidence for the Big Bang: 2.7 K microwave radiation.
- Photon density $\sim 400/$cm$^3$
Early history of the Universe

![Graph showing the temperature evolution of the Universe over time.](image)
Evolution of the Universe

- $10^{-43}$ s, Planck time $10^{32}$ K, $vol = 10^{-31}$
  $vol_{current}$
- $k_B T (eV) = 8.5 \times 10^{-5} T (K)$
- Matter is QGP, all particle present.
- $10^{-6}$ s, $T \sim 10^{13}$K, hadronic matter condenses out.
- Matter is nucleons, mesons, neutrinos, photons, electrons.
Evolution of the Universe

- \(10^{-2} \text{ s. } T \sim 10^{11} \text{ K, } \rho \sim 4 \times 10^6 \text{ kg/m}^3\)

\[
T(K) = \frac{1.5 \times 10^{10}}{\sqrt{t(s)}}
\]

\[\nu_e + p \iff e^+ + n\]

\[\nu_e + n \iff p + e^-\]

\[n/p = \exp(-\Delta mc^2/kT)\]
Evolution of the Universe

- At $T=10^{12}K$, $n/p \sim 1$, at $T=10^{11}K$ $n/p \sim 0.86$, etc. At $T = 10^{11}K$, no complex nuclei were formed because the temperature was too high to allow deuterons to form. When the temperature fell to $T=10^{10}K$ ($t \sim 1$ s), the creation of $e^+/e^-$ pairs (pair production) ceased because $kT < 1.02$ MeV and the neutron/proton ratio was $\sim 17/83$. At a time of 225s, this ratio was $13/87$, the temperature was $T \sim 10^9K$, then density was $\sim 2 \times 10^4$kg/m$^3$, and the first nucleosynthetic reactions occurred.
First Nucleosynthesis

- **Hydrogen burning**
  - \( n + p \rightarrow d + \gamma \)
  - \( p + d \rightarrow ^{3}\text{He} + \gamma \)
  - \( n + d \rightarrow ^{3}\text{H} + \gamma \)

- **He formation**
  - \( ^{3}\text{H} + p \rightarrow ^{4}\text{He} + \gamma \)
  - \( ^{3}\text{He} + n \rightarrow ^{4}\text{He} + \gamma \)
  - \( ^{3}\text{H} + d \rightarrow ^{4}\text{He} + n \)
  - \( d + d \rightarrow ^{4}\text{He} + \gamma \)
After about 30 m, nucleosynthesis ceased. The temperature was $\sim 3 \times 10^8$K and the density was $\sim 30$ kg/m$^3$. Nuclear matter was 76% by mass protons, 24% alpha particles with traces of deuterium, $^3$He and $^7$Li. The $\gamma/n/p$ ratio is $10^9/13/87$. The relative ratio of $p/^4$He/$d/^3$He/$^7$Li is a sensitive function of the baryon density of the Universe. Chemistry began about $10^6$ years later, when the temperature had fallen to 2000K and the electrons and protons could combine to form atoms. Further nucleosynthesis continues to occur in the interiors of stars.
Stellar Nucleosynthesis

• All elements beyond H and He synthesized in stellar interiors
• Stellar nucleosynthesis continues to date \((2 \times 10^5 \text{ y})\) \(^{99}\text{Tc}\) lines in stars)
Stellar Evolution

- Population III stars (protostars) --
  H, He, short lifetimes, now extinct
- Population II stars (H, He, 1% heavier elements)
- Population I stars (H, He, 2-5% heavier elements) Includes our sun.
Sun

- Typical Population I star.
- mass=$2 \times 10^{30}$ kg
- radius=$7 \times 10^6$ m
- $\rho\approx1.41 \times 10^3$ kg/m$^3$
- surface $T \sim 6000$K
- Luminosity $\sim 3.83 \times 10^{26}$ W
- age $\sim 4.5 \times 10^9$ y.
Herzsprung-Russell Diagrams

![Herzsprung-Russell Diagram](image)
Stellar evolution and H-R diagrams
Aside on our Sun

• \(\sim 7 \times 10^9\) more years on main sequence

• \(1.1-1.5 \times 10^9\) years, luminosity will increase by \(\sim 10\%\), making Earth uninhabitable.

• Terrestrial life has used up about \(3/4\) of its lifespan.
Supernovas

- Massive stellar explosions
- \( \sim 10^{51} \) ergs released in a few seconds
- 2-3/century, last observation was 1987.
- Some supernovas lead to the formation of neutron stars.
Fig. 7 The Crab Nebula (M 1). The nebula consists of matter ejected in a supernova explosion. The material is spread over a volume of 10 light years in diameter and is still expanding at velocities of ≈1800 km/s. Its distance from Earth is about 6000 light years. The supernova explosion was detected on July 4, 1954, by Chinese astronomers. It is one of the very few historically observed supernovae in our Galaxy. The remnant of the supernova, located in the middle of the nebula, is a neutron star that spins with a period of ≈30 ms (pulsar).

The presence of a remnant neutron star and of hydrogen in the ejecta supports the association of the Crab Nebula with a type II supernova. The image is a three color composite. The green light is predominantly produced by hydrogen emission from material that was ejected by the exploding star. The blue light arises mainly from relativistic electrons that spiral in a large-scale magnetic field (synchrotron radiation) and that are continuously ejected from the rapidly spinning neutron star. Credit: NASA, ESA, and J. Hester (Arizona State University).
Fig. 6 Supernova 1987A in the Large Magellanic Cloud (a nearby small galaxy that is a satellite of our Galaxy) was the brightest exploding star seen in 400 years. Its distance from Earth is ≈160,000 light years. The supernova was of type II and its progenitor was a massive star (a blue supergiant). The shock wave from the supernova has been moving toward a ring of matter, about two light years across, that was probably ejected by the central star about 20,000 years before the explosion. The image shows many hot spots that are created by the supernova shock compressing and heating the gas of the ring. (The brightest spot on the lower right side of the ring is a star that happens to lie along the Hubble Space Telescope’s line of sight). The elongated and expanding object in the middle of the ring is the debris from the explosion. Credit: NASA, P. Challis, R. Kirshner (Harvard-Smithsonian Center for Astrophysics) and B. Sugerman (STScI).
Figure 7.11. Birth of a neutron star and supernova remnant.
Thermonuclear reaction rates

\[ R = N \sigma \phi \]

\[ R = N_x N_y \int_0^\infty \sigma(v)vdv = N_x N_y \langle \sigma v \rangle \]

\[ R = \frac{N_x N_y \langle \sigma v \rangle}{1 + \delta_{xy}} \]

\[ P(v) = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} \]

\[ \langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E)E \exp \left( -\frac{E}{kT} \right) dE \]
But these are charged particle reactions!

- For p + p, CB \approx 550 keV.

- kT \approx 1.3 keV \rightarrow\text{barrier penetration problem}
\[ P = \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{h\nu}\right) = \exp\left(-31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}\right) \]

\[ \sigma(E) = \frac{1}{E} \exp\left(-31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}\right) S(E) \]

\[ \langle \alpha \rangle = \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{3/2}}\right] dE \]
Stellar Nucleosynthesis--A Scorecard

• Big Bang $\rightarrow$ 75% H, 25% He, trace $^7$Li

• From $\sim 10^6$ years after the Big Bang to present, get nuclear fusion reactions in stars that synthesize the elements up to $A \sim 60$. 
<table>
<thead>
<tr>
<th>Fuel</th>
<th>T(K)</th>
<th>kT(MeV)</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1H$</td>
<td>$5 \times 10^7$</td>
<td>0.002</td>
<td>$^4He$</td>
</tr>
<tr>
<td>$^4He$</td>
<td>$2 \times 10^8$</td>
<td>0.02</td>
<td>$^{12}C, ^{16}O, ^{20}Ne$</td>
</tr>
<tr>
<td>$^{12}C$</td>
<td>$8 \times 10^8$</td>
<td>0.07</td>
<td>$^{16}O, ^{20}Ne, ^{24}Mg$</td>
</tr>
<tr>
<td>$^{16}O$</td>
<td>$2 \times 10^9$</td>
<td>0.2</td>
<td>$^{20}Ne, ^{28}Si, ^{32}S$</td>
</tr>
<tr>
<td>$^{20}Ne$</td>
<td>$1.5 \times 10^9$</td>
<td>0.13</td>
<td>$^{16}O, ^{24}Mg$</td>
</tr>
<tr>
<td>$^{28}Si$</td>
<td>$3.5 \times 10^9$</td>
<td>0.3</td>
<td>A &lt; 60</td>
</tr>
</tbody>
</table>
Hydrogen Burning

• First stage of a star; converts H into He.
• First reaction (pp):  \( p + p \rightarrow d + e^+ + \nu_e \)  \( Q = 0.42 \text{ MeV} \)
• Weak branch (pep)  \( p + e^- + p \rightarrow d + \nu_e \)  \( Q = 1.42 \text{ MeV} \)
• Next Reaction  \( d + p \rightarrow ^3\text{He} + \gamma \)  \( Q = 5.49 \text{ MeV} \)
• 86% Branch  \( ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p \)  \( Q = 12.96 \text{ MeV} \)
• Net reaction
  •  \( 4p \rightarrow ^4\text{He} + 2e^+ + 2\nu_e \)  \( Q = 26.7 \text{ MeV} \)
Hydrogen Burning (ppI chain)
Side reaction

- $^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \nu_e$
- $\text{e}^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e \quad Q = 0.86 \text{ MeV}$
- $p + ^7\text{Li} \rightarrow 2^4\text{He}$

- This side branch along with the $p+p, d+p$ is called the ppII process
Another side branch

- $^7\text{Be} + p \rightarrow ^8\text{B} + \gamma$
- $^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e$
- $^8\text{Be}^* \rightarrow 2^4\text{He}$

- This sequence along with the p+p, d+p, etc is called the PPIII chain.
\[ p (p, e^+ \nu) d \]
\[ d (p, \gamma) ^3He \]
\[ ^3He (p, 2p) ^4He \]
\[ ^3He (\alpha, \gamma) ^7Be \]
\[ ^7Be (\beta^-) ^7Li \]
\[ ^7Li (p, \alpha) ^4He \]
\[ ^7Be (p, \gamma) ^8B \]
\[ ^8B (\beta^+) ^8Be^* \]
\[ ^8Be^* (\alpha) \alpha \]

Chain I
\[ Q_{\text{effective}} = 26.20 \text{ MeV} \]

Chain II
\[ Q_{\text{effective}} = 25.66 \text{ MeV} \]

Chain III
\[ Q_{\text{effective}} = 19.17 \text{ MeV} \]
CNO Cycle

\[ {^{12}\text{C}} + \text{p} \rightarrow {^{13}\text{N}} + \gamma \]
\[ {^{13}\text{N}} \rightarrow {^{13}\text{C}} + \text{e}^+ + \nu_e \]
\[ {^{13}\text{C}} + \text{p} \rightarrow {^{14}\text{N}} + \gamma \]
\[ {^{14}\text{N}} + \text{p} \rightarrow {^{15}\text{O}} + \gamma \]
\[ {^{15}\text{O}} \rightarrow {^{15}\text{N}} + \text{e}^+ + \nu_e \]
\[ {^{15}\text{N}} + \text{p} \rightarrow {^{12}\text{C}} + {^4\text{He}} \]

Net effect is \( 4\text{p} \rightarrow {^4\text{He}} + 2\text{e}^+ + 2\nu_e \)
CNO Cycle
Relative Importance of pp and CNO cycle
He burning

- Eventually the H fuel will be exhausted, get gravitational collapse, further heating and red giant formation. Then He burning will commence. The reaction is the $3\alpha$ process.

$$3 \, ^4\text{He} \rightarrow ^{12}\text{C} \quad Q = 7.37 \, \text{MeV}$$
\[ p_f = 10^5 \text{ g/cm}^3 \]
\[ x_a = 1 \]
\[ ^{12}\text{C} + ^{12}\text{C} \rightarrow ^{20}\text{Ne} + ^{4}\text{He} \]
\[ ^{12}\text{C} + ^{12}\text{C} \rightarrow ^{23}\text{Na} + \text{p} \]
\[ ^{12}\text{C} + ^{12}\text{C} \rightarrow ^{23}\text{Mg} + \text{n} \]
\[ ^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg} + \gamma \]
\[ ^{16}\text{O} + ^{16}\text{O} \rightarrow ^{24}\text{Mg} + 2 ^{4}\text{He} \]
\[ ^{16}\text{O} + ^{16}\text{O} \rightarrow ^{28}\text{Si} + ^{4}\text{He} \]
\[ ^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{P} + \text{p} \]
\[ ^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{S} + \text{n} \]
\[ ^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S} + \gamma \]
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>H burning</td>
<td>$6 \times 10^9$ years</td>
</tr>
<tr>
<td>He burning</td>
<td>$0.5 \times 10^6$ years</td>
</tr>
<tr>
<td>C burning</td>
<td>200 years</td>
</tr>
<tr>
<td>Ne burning</td>
<td>1 year</td>
</tr>
<tr>
<td>O burning</td>
<td>Few months</td>
</tr>
<tr>
<td>Si burning</td>
<td>days</td>
</tr>
</tbody>
</table>
Synthesis of $A > 60$

- Use neutron capture
- There are two types of $n$-capture reactions. one on a slow time scale, the \textit{s}-process and one on a rapid time scale, the \textit{r} process.
- \textit{In \textit{s} process reactions,} $\beta^-$ decay intervenes between $n$ captures while in the \textit{r} process, it does not.
s-Process

• Example
  - $^{56}\text{Fe} + n \rightarrow ^{57}\text{Fe} \text{ (stable)} + \gamma$
  - $^{57}\text{Fe} + n \rightarrow ^{58}\text{Fe} \text{ (stable)} + \gamma$
  - $^{58}\text{Fe} + n \rightarrow ^{59}\text{Fe} \left( t_{1/2} = 44.5 \text{ d} \right) + \gamma$
  - $^{59}\text{Fe} \rightarrow ^{59}\text{Co} \text{ (stable)} + \beta^- + \nu_e$

• Process terminates at $^{209}\text{Bi}$

\[
^{209}\text{Bi}(n,\gamma)^{210}\text{Bi} \xrightarrow{\beta^-} ^{210}\text{Po} \xrightarrow{\alpha} ^{206}\text{Pb} \xrightarrow{3(n,\gamma)} ^{209}\text{Pb} \xrightarrow{\beta^-} ^{209}\text{Bi}
\]
r-process (in supernovas)
p process

• makes proton-rich nuclei
• most reactions are photonuclear reactions like $(\gamma, p)$, $(\gamma, n)$, $(\gamma, \alpha)$
• probably occurs in supernovas.
rp process

\[ T_s = 1.5 \]
\[ \rho = 10^6 \text{g/cm}^3 \]
Solar Neutrinos

Sun emits $1.8 \times 10^{38}$ neutrinos/s with the flux hitting the Earth being $6.4 \times 10^{10}$ neutrinos/s/cm$^2$.

Table 12.3 Predicted solar neutrino fluxes (Bahcall and Pena-Garay)

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux(particles/s/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>$5.94 \times 10^{10}$</td>
</tr>
<tr>
<td>$pep$</td>
<td>$1.40 \times 10^{8}$</td>
</tr>
<tr>
<td>$hep$</td>
<td>$7.88 \times 10^{3}$</td>
</tr>
<tr>
<td>$^7Be$</td>
<td>$4.86 \times 10^{7}$</td>
</tr>
<tr>
<td>$^8B$</td>
<td>$5.82 \times 10^{6}$</td>
</tr>
<tr>
<td>$^{13}N$</td>
<td>$5.71 \times 10^{8}$</td>
</tr>
<tr>
<td>$^{15}O$</td>
<td>$5.03 \times 10^{8}$</td>
</tr>
<tr>
<td>$^{17}F$</td>
<td>$5.91 \times 10^{6}$</td>
</tr>
</tbody>
</table>
Fluxes and Detectors
Solar Neutrino Detectors

- The most famous detector is the Chlorine detector of Ray Davis.
- Contained 100,000 gal $C_2Cl_4$ in a cavern 1600 m below the earth’s surface in the Homestake mine.
- Reaction used:
  \[ \nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^- \]
  
  - Ar nuclei collected as gas, detect 2.8 keV $e$ from Auger from EC
  - Produce ~3 atoms/wk in volume of $10^{30}$ atoms.
Gallex detector

- $\nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^-$
- Collect Ge gas product
- threshold = 0.232 MeV
Super K

- Detect Cerenkov radiation from
  \[ \nu + e^- \rightarrow \nu + e^- \]
- threshold = 8 MeV
SNO

- $\nu + e^- \rightarrow \nu + e^-$
- $\nu_e + d \rightarrow 2p + e^-$
- $\nu + d \rightarrow n + p + \nu$
- Sensitive to all types of neutrinos
Results

• Davis: measured $2.1 \pm 0.3$ SNU
  expect $7.9 \pm 2.4$ SNU

$1 \text{ SNU} = 10^{-36}$ neutrino captures/s/target atom

• Gallex: measured $77 \pm 10$ SNU
  expect $127$ SNU
Total Rates: Standard Model vs. Experiment
Bahcall-Pinsonneault 2000

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Theory:
- $^7\text{Be}$
- $p-p$, pep
- $^8\text{Be}$
- CNO

Experiments:
- SNO $\nu_e$

Uncertainties:
Solution

• neutrino oscillations
• imply neutrino has mass
Synthesis of Li, Be, B