Lesson 16

Interaction of Radiation with Matter--Uncharged Particles
Introduction

• This lesson deals with the interaction of photons and neutrons with matter

• Photon interactions are atomic interactions (with the electrons)

• Neutrons interact via nuclear processes
Electromagnetic Radiation

• Results of interaction is the production of energetic electrons

• No concept of range, just exponential attenuation

\[ I = I_0 e^{-\mu x} \]
Absorption of photons

\[ I = I_0 e^{-\mu x} \]

\( \mu \) and \( x \) need to be in compatible units

If \( x \) is expressed as a linear distance (cm), then
\( \mu \) is expressed in cm\(^{-1}\), the linear absorption coefficient

If \( x \) is expressed in areal density units (g/cm\(^2\)), then
\( \mu \) is expressed in cm\(^2\)/g, the mass absorption coefficient
Mechanisms of photon interaction with matter

- There are three basic mechanisms for the interaction of photons with matter, the **photoelectric effect** (low energies, high Z), **Compton scattering** (all energies, all Z) and **pair production** ($E_\gamma > 1.02$ MeV, high Z)
Mechanisms of photon interaction with matter

- **Low energy** photon interacts with an atom, emitting a photoelectron.
- **Medium energy** photon results in a recoil electron.
- **High energy** (E > 1.02 MeV) photon causes electron-positron annihilation.
Photoelectric Effect

- photon $\rightarrow$ electron
- $E_e = E_\gamma - B.E.$
- leaves vacancy in electron shells, get X-rays and Auger electrons to fill vacancies
- overall probability

\[ \sigma \propto \frac{Z^{4-5}}{E_\gamma^{7/2}} \]
Critical absorption

K,L,M binding energies
88, 15, 3 keV

“Graded” shielding
Compton scattering
Derivations (from Sarantities)

We need to use the following relativistic equation to relate energy to rest mass and momentum.

\[ E_e = (E_0^2 + p_e^2 c^2)^{1/2} = \left[(m_0 c^2)^2 + p_e^2 c^2\right]^{1/2}. \quad (4.17) \]

where \( E_e \) is the total electron energy, \( E_0 \) is the electron rest energy, \( m_0 \) is the electron rest mass, and \( p \) the electron momentum.

**Energy conservation:**

\[ E_\gamma^0 + E_0 = E_\gamma + (E_0^2 + p_e^2 c^2)^{1/2}. \quad (4.18) \]

**Conservation of parallel momentum** \( p_\parallel \):

\[ \frac{E_\gamma^0}{c} = \frac{E_\gamma}{c} \cos \theta + p_e \cos \phi. \quad (4.19) \]

**Conservation of perpendicular momentum** \( p_\perp \):

\[ \frac{E_\gamma}{c} \sin \theta = p_e \sin \phi. \quad (4.20) \]
The last equation can be written

\[ \cos \phi = (1 - \sin^2 \phi)^{1/2} = \left[ 1 - \left( \frac{E_\gamma}{p_c c} \sin \theta \right)^2 \right]^{1/2} \]

where we substituted \( \sin \phi \) using Eq. 4.20.
Next we substitute \( \cos \phi \) into Eq. 4.19 to obtain:

\[ \frac{E_\gamma^0}{c} = \frac{E_\gamma}{c} \cos \theta + p_c \left[ 1 - \left( \frac{E_\gamma}{p_c c} \sin \theta \right)^2 \right]^{1/2} \]

By multiplying by \( c \)

\[ E_\gamma^0 = E_\gamma \cos \theta + \left[ (p_c c)^2 - E_\gamma^2 \sin^2 \theta \right]^{1/2} \]

and solving the last expression for \((p_c c)^2\) and expanding:

\[ (p_c c)^2 = (E_\gamma^0)^2 + (E_\gamma)^2 \cos^2 \theta - 2E_\gamma^0 E_\gamma \cos \theta + (E_\gamma)^2 \sin^2 \theta \]

\[ = (E_\gamma^0)^2 + (E_\gamma)^2 - 2E_\gamma^0 E_\gamma \cos \theta. \]

Now substitute this expression for \((p_c c)^2\) in Eq. 4.18:

\[ E_\gamma^0 + E_0 = E_\gamma + \left[ (E_0)^2 + (E_\gamma^0)^2 + (E_\gamma)^2 - 2E_\gamma^0 E_\gamma \cos \theta \right]^{1/2} \]

Squaring the latter Equation we find:

\[ (E_\gamma^0 + E_0 - E_\gamma)^2 = (E_0)^2 + (E_\gamma^0)^2 + (E_\gamma)^2 - 2E_\gamma^0 E_\gamma \cos \theta \]

Now expand the square:

\[ (E_0)^2 + (E_\gamma^0)^2 + (E_\gamma)^2 + 2E_\gamma^0 E_\gamma - 2E_\gamma^0 E_\gamma - 2E_\gamma E_\gamma \cos \theta \]

\[ = (E_0)^2 + (E_\gamma^0)^2 + (E_\gamma)^2 - 2E_\gamma^0 E_\gamma \cos \theta \]

This simplifies to give:

\[ E_\gamma^0 E_0 - E_\gamma^0 E_\gamma - E_\gamma E_\gamma = -E_\gamma^0 E_\gamma \cos \theta \]

Finally, by dividing by \( E_\gamma^0 E_\gamma E_0 \) we get

\[ \frac{1}{E_\gamma} - \frac{1}{E_\gamma^0} = \frac{1}{E_0} (1 - \cos \theta). \]  

(4.21)

This is the famous Compton scattering formula, originally written after the substitution \( E = h \nu = \frac{h c}{\lambda} \) as:

\[ \lambda - \lambda^0 = \frac{h}{m_e c} (1 - \cos \theta). \]
Understanding the results

\[ E_{\gamma}^{\text{min}} = \frac{m_e c^2}{2} \left( \frac{1}{1 + \frac{m_e c^2}{2E_{\gamma}}} \right) \approx 255\text{keV} \]

\( \theta = 180 \)
Pair production

- $E_\gamma > 1.02 \text{ MeV}$
- photon $\rightarrow$ electron and positron
- cross section increases roughly as $Z^2 \ln(E_\gamma / mc^2)$
- positron annihilation

\[ e^- + e^+ \rightarrow \gamma + \gamma \]
Overview

![Graph showing the relationship between Z of absorber and hν in MeV, with data points for photoelectric, Compton, and pair production effects.](image-url)
Neutrons

- Neutron has small magnetic moment, does not interact significantly with electrons.
- Stopping is by neutron-nucleus interaction.
- Exponential attenuation:

\[ I = I_0 e^{-\mu_E x} \]

\[ \mu_E = \frac{1}{\lambda_E} = N_0 \sigma_{Total}(E) \]

\[ \sigma_{average} = (f_1 \sigma_{Total}(E)_1 + f_2 \sigma_{Total}(E)_2 + f_3 \sigma_{total}(E)_3 + \ldots ) \]
Mechanisms of neutron-nucleus interaction

• Elastic scattering
• Inelastic scattering
• Radiative capture
• Fission
• Knock-out reactions

\[ \sigma_{\text{total}} = \sigma_{\text{elastic}} + \sigma_{\text{inelastic}} + \sigma_{\text{capture}} + \cdots \]
Kinematics of neutron scattering

In the center of mass (cm) system, we have

\[ V_1 = v_1 - v_0 \]
\[ V_2 = v_0 \]
\[ m_1 \rightarrow m_2 \]
\[ m_1, v_0 \rightarrow m_2, v_2' \]

After the collision, the relationship between the cm and the lab systems is

\[ v_0 = \frac{M_1}{M_1 + M_2} v_1 = \text{velocity of cm} \quad (17.43) \]
Derivation

\[ T'_1 = \frac{1}{2} m_1 (v_1')^2 \]

\[ T'_1 = \frac{1}{2} m_1 (V_1 + v_0)^2 = \frac{1}{2} m_1 (V_1^2 + v_0^2 + 2V_1v_0 \cos \theta) \]

\[ T_1(\text{max}) = \frac{1}{2} m_1 (V_1^2 + v_0^2 + 2V_1v_0) = \frac{1}{2} m_1 v_1^2 = T_1 \]

\[ T_1(\text{min}) = \frac{1}{2} m_1 (V_1^2 + v_0^2 - 2V_1v_0) = \frac{1}{2} m_1 (V_1 - v_0)^2 \]

\[ = \frac{1}{2} m_1 (v_1 - 2v_0)^2 \]

\[ = T_1 \left( \frac{m_1 - m_2}{m_1 + m_2} \right)^2 \]

**If m1=m2**

\[ T_1(\text{min}) = 0 \]

\[ T_1(\text{max}) = T_1 \]

\[ T'_1 = \frac{T_1 (1 + \cos \theta)}{2} \]
Derivation (cont)

Single scattering

Multiple scattering

\[ T_1(\text{average}) \approx \left( \frac{1}{2} \right)^n T_1 \]
Radiation Safety

• Want to estimate amount of energy transferred to tissue and its effect.

• Units of energy deposit (exposure)
  • Roentgen--The amount of radiation that produces 1 esu of ions in 1 cm$^3$ of dry air at STP = 2.58E-04 Coulomb/kg dry air
  • Rad = the dose that deposits 100 ergs/g of material
  • Gray = the dose that produces 1 Joule/Kg
Relative Biological Effectiveness (RBE)

- RBE = relative efficiency for producing a biological effect
- RBE $\propto$ LET
- RBE also called radiation weighting factor (Table 17-3)
Dose

• Equivalent dose = dose (in grays) \times (radiation weighting factor)

• Unit of equivalent dose = Sievert (Sv)

• Old unit (rem=roentgen-equivalent-man)

• 1Sv=100 rem; 10\mu Sv=1mrem